AP Calculus AB – Summer Assignment 2020

Due Aug 20, 2020 – pending any changes to our School Year start date of Aug 17th.

Good Luck, Do Your Best, & Stay Safe!!!

Part 1 – Free Response

Show ALL work for the following problems – neatly on your own paper, submit your answers, and a picture of your work in Google Classroom.

1. Solve $2x^2 + 5x = 12$ using three different methods. Simplify your answer(s) completely. Name and Label each method.

2. Solve $3x^{(2/3)} = 24$. Simplify your answer(s) completely.

3. The expression $-3(x - 2)^2 - 5$ represents a parabola. Does the parabola have a maximum or a minimum? Explain how you know. What are the coordinates of the maximum or minimum? State the domain and range of the parabola.

4. Solve $3e^{5x + 1} = 7$.

5. State the Amplitude, Phase Shift, and Period of $\cos\left(x - \frac{1}{2}\pi\right)$.

6. Solve $2 \log_3 (x + 1) = 4$.

7. Simplify $\dfrac{x+1}{2-\dfrac{1}{x+1}}$ completely.

8. Factor & Solve $5x^5+15x^4-20x^3=0$.

9. Solve $\frac{3}{4}-\frac{1}{6x}=\frac{1}{x}$.

10. Let $f(x) = x^2 - 3x + 4$. Evaluate $f(x+1)$.

11. Use Long Division to simplify

$$(2x^3-5x^2+5x-6)\div(x-2).$$

Part 2 – Multiple Choice

1. Determine $(0,4]\cap[-2,1)$.

A. $[0,1]$
B. $[-2,0)\cup(0,1)\cup(1,4]$
C. $(0,1)$
D. $(-\infty,0)\cup(0,1)\cup(1,\infty)$
E. $[-2,4]$

2. The expression $\ln e - \ln x^3$ is equivalent to

A. $-\ln(x^3)$
B. $\ln(e)+\ln(-x^3)$
C. $\frac{1}{\ln(x^3)}$
D. $\frac{\ln(e)}{\ln(x^3)}$
E. $1-\ln(x^3)$

3. Simplify $\frac{3+3\sqrt{6}}{6}$

A. $\frac{1+\sqrt{2}}{2}$
B. $\frac{\sqrt{6}}{2}$
C. $\frac{1+\sqrt{6}}{2}$
D. $\frac{3\sqrt{6}}{2}$
E. $\frac{1+3\sqrt{6}}{2}$

4. For what values of x in the interval $[-2\pi, 2\pi]$ does the graph of $y = \cot(2x)$ have vertical asymptote? (Angles are measured in radians)

a. $-2, -1, 0, 1, 2$

b. $-2\pi, -\frac{3\pi}{2}, -\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

c. $-2\pi, -\pi, 0, \pi, 2\pi$

d. $-\frac{3\pi}{2}, \frac{3\pi}{2}$

e. $-2\pi, 0, 2\pi$

5. The general solution of the equation $\cos(2\theta) = 1$ is

a. $k\pi$, where k is an integer
b. 0
c. $2k\pi$, where k is an integer
d. $\frac{\pi}{2} + 2k\pi$, where k is an integer
e. $\frac{3\pi}{2} + 2k\pi$, where k is an integer

6. Simplify

$$\sec x - \sec x \,.\sin^2 x$$

a. 1
b. $\sec x$
c. $\sin^2 x$
d. $\cos^2 x$
e. $\cos x$

7. Which of the following is an expression for $\cos(2\alpha)$

a. $1 + 2\cos^2(\alpha)$
b. $-1 + 2\cos^2(\alpha)$
c. $1 - \cos^2(\alpha)$
d. $-1 - \cos^2(\alpha)$
e. $2\cos(\alpha)$

8. By using sum or difference formulas, $\cos(\frac{\pi}{2} - x)$ can be written as

a. $-\cos x$

b. $\sin x$

c. $\cos x$

d. $-\sin x$

e. None of the above

9. Find $\frac{f(2+h)-f(2)}{h}$ if $f(x) = x^2 + 2x - 1$

a. $h + 6$

b. 2

c. $2 + h$

d. $h^2 + 2h - 1$

e. 1

10. Which of the following statements are true?

I. $(\ln x)^2 = 2 \ln x$

II. $\log_4(3x^4) = 4 \log_4(3x)$

III. $\log(x - y) = \frac{\log x}{\log y}$

IV. $\log_3 \frac{9}{4} = 2 - \log_3 4$

V. $\ln(x^2) = 2 \ln x$

a. I and II only

b. I, II, and III only

c. I and III only

d. IV and V only

e. I and IV only

11. Find the horizontal asymptote (HA) and vertical asymptote (VA) of

$$f(x) = \frac{x^2 - 4}{x(x+2)}$$

a. HA: $y = 1$ VA: $x = 0, x = -2$

b. HA: $y = 0$ VA: $x = 0, x = -2$

c. HA: $y = 1$ VA: $x = 0$

d. HA: $y = 0$ VA: $x = 0$

e. HA: None VA: $x = 0, x = -2$

12. Find the value of $f(2) - f(0)$, if

$$f(x) = \begin{cases} 2 - x, & x < 1 \\ x^2 - x + 1, & x \geq 1 \end{cases}$$

a. 3

b. -1

c. 2

d. 0

e. 1

Part 3 – Multiple Choice

1. Find the slope and y-intercept of the line that is parallel to $2x + 3y = 5$ and passes through the point $(1, -1)$

   a. $Slope = \frac{2}{3}; y - intercept = \frac{5}{3}$

   b. $Slope = -\frac{2}{3}; y - intercept = \frac{1}{3}$

   c. $Slope = -\frac{2}{3}; y - intercept = -\frac{1}{3}$

   d. $Slope = -\frac{2}{3}; y - intercept = \frac{5}{3}$

   e. None of the above

2. The domain of the function $f(x) = \sqrt{2 - x}$ is the set of numbers x satisfying:
   (A) $x \le 2$ (B) $x \ne 2$ (C) $x \ge 2$ (D) all real numbers

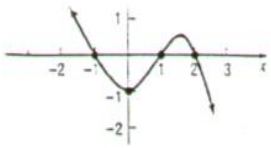
3. Consider the functions $f(x) = x^3 + x^2$ and $g(x) = x^2 + 1$. Then
   (A) f and g are both even (B) f is odd and g is even
   (C) f is neither even nor odd and g is even (D) f and g are neither even nor odd

4. Write the expression for the function whose graph is the graph of $y = x^3$ but shifted down 4 units and left 5 units.
   (A) $y = (x - 5)^3 - 4$ (B) $y = (x + 5)^3 - 4$ (C) $y = (x - 5)^3 + 4$ (D) $y = (x - 4)^3 - 5$

5. Given $f(x) = 2x + 3$ and $g(x) = \sqrt{x}$, find $(f \circ g)(x)$.
   (A) $(f \circ g)(x) = \sqrt{2x + 3}$ (B) $(f \circ g)(x) = (2x + 3)\sqrt{x}$
   (C) $(f \circ g)(x) = 2\sqrt{x} + 3$ (D) $(f \circ g)(x) = 2\sqrt{x + 3}$

6. Find the vertex V and the x-intercepts x_1 and x_2 of the quadratic function $f(x) = 2x^2 - 8x$.
   (A) V(2, -8); $x_1 = 0$, $x_2 = 4$ (B) V(2, -4); $x_1 = 0$, $x_2 = 4$
   (C) V(0, 0); $x_1 = -2$, $x_2 = 4$ (D) V(0,0); $x_1 = 2$, $x_2 = -8$

7. Which of the following functions might have the graph pictured here?



   (A) $y = \frac{1}{2}(x^2 - 1)(x - 2)$ (B) $y = \frac{1}{2}(x^2 + 1)(x - 2)$
   (C) $y = (x^2 - 1)\left(1 - \frac{x}{2}\right)$ (D) $y = -(x - 1)(x - 2)$

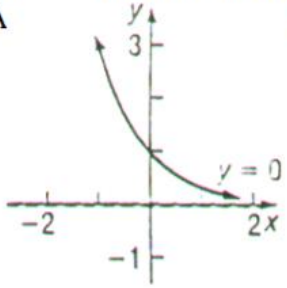
8. Find the vertical and horizontal asymptotes of the function $f(x) = \dfrac{2x}{x - 5}$.

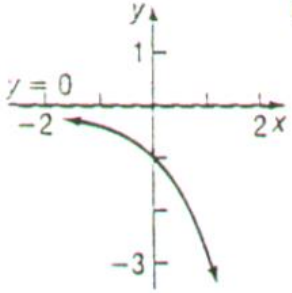
(A) Vertical: $x = 5$; Horizontal: $y = 0$ (B) Vertical: $x = 2$; Horizontal: $y = 1$
(C) Vertical: $x = 2$; Horizontal: $y = 5$ (D) Vertical: $x = 5$; Horizontal: $y = 2$

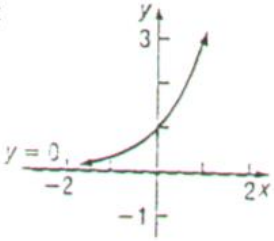
9. Find the inverse of the function $f(x) = \dfrac{1}{x - 2}$.

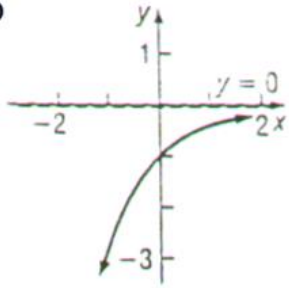
(A) $f^{-1}(x) = x - 2$ (B) $f^{-1}(x) = \dfrac{1}{x} + 2$ (C) $f^{-1}(x) = \dfrac{1}{x + 2}$ (D) $f^{-1}(x) = x + 2$

10. Which of the following is the graph of the function $f(x) = -3^{-x}$?

A 

B 

C 

D 

11. Give the domain D, range R, and the x-intercept X of the function $y = \ln x$.
(A) $D = \{x \mid x > 0\}$; R = all real numbers; $X = 1$
(B) D = all real numbers; $R = \{y \mid y > 0\}$; $X = 1$
(C) $D = \{x \mid x > 0\}$; $R = \{y \mid y > 0\}$; $X = 1$
(D) D = all real number; R = all real numbers; $X = 0$

12. Solve the equation: $\log_4 x + \log_4 (x - 3) = 1$.
(A) $x = 4$ or $x = 1$ (B) $x = 3.5$ (C) $x = 4$ (D) no solution

13. Iodine 131 is a radioactive material that decays according to the function $A(t) = A_0 e^{-0.087t}$, where A_0 is the initial amount present and A is the amount present at time t (in years). Determine how long it takes for 250 grams of iodine 131 to decay to 50 grams.
(A) 19 years (B) 5 years (C) 50 years (D) 10 years

14. Find the length of the arc of a circle with radius 10 cm subtended by a central angle of 45°.
(A) 450 cm (B) $\dfrac{40}{\pi}$ cm (C) $\dfrac{\pi}{40}$ cm (D) $\dfrac{5\pi}{2}$ cm

15. Find $\sin\dfrac{\pi}{6}$ and $\tan\dfrac{\pi}{3}$: (A) $\dfrac{1}{2}, \sqrt{3}$ (B) $\dfrac{\sqrt{3}}{2}, \sqrt{3}$ (C) $\dfrac{1}{2}, \dfrac{\sqrt{3}}{3}$ (D) $\dfrac{\sqrt{3}}{2}, \dfrac{\sqrt{3}}{3}$

16. Suppose that $\cos\theta = 3/5$ and θ lies in Quadrant IV. Find $\sin\theta$ and $\tan\theta$.

(A) $\sin\theta = 4/5$, $\tan\theta = -4/3$ (B) $\sin\theta = -4/5$, $\tan\theta = -4/3$

(C) $\sin\theta = 4/5$, $\tan\theta = 4/3$ (C) $\sin\theta = -4/5$, $\tan\theta = 4/3$

17. Determine the equation of the sine function which has amplitude 2 and period 4.

(A) $y = 2\sin(4x)$ (B) $y = 2\sin\left(\frac{\pi}{2}x\right)$ (C) $y = 4\sin(2x)$ (D) $y = 4\sin\left(\frac{\pi}{4}x\right)$

18. For what values of x between 0 and 2π does $y = \sec x$ have vertical asymptotes?

(A) $\frac{\pi}{2}, \frac{3\pi}{2}$ (B) $\frac{\pi}{4}, \frac{3\pi}{4}$ (C) $0, \pi, 2\pi$ (D) There are no vertical asymptotes.

19. Find the exact value of $\tan^{-1}(-1)$ and $\cos^{-1}(-1)$.

(A) $\frac{3\pi}{4}, \pi$ (B) $\frac{\pi}{4}, 0$ (C) $\frac{3\pi}{4}, \frac{3\pi}{2}$ (D) $\frac{-\pi}{4}, \pi$

20. Which of the following equals $1 - \frac{\sin^2\theta}{1 - \cos\theta}$?

(A) $\cos\theta$ (B) $-\cos\theta$ (C) $1 - \sin\theta$ (D) $1 + \sin\theta$

21. If $\sin\theta = \frac{1}{3}$ and θ lies in Quadrant II, find the exact value of $\sin\left(\theta + \frac{\pi}{6}\right)$.

(A) $\frac{5}{6}$ (B) $\frac{\sqrt{3} + \sqrt{8}}{6}$ (C) $\frac{\sqrt{3} - \sqrt{8}}{6}$ (D) $\frac{\sqrt{3} - 1}{2}$

22. If $\cos\theta = \frac{-3}{5}$ and $\pi < \theta < \frac{3\pi}{2}$, then find $\cos\left(\frac{\theta}{2}\right)$.

(A) $\frac{-3}{10}$ (B) $\frac{\sqrt{5}}{5}$ (C) $\frac{-2\sqrt{5}}{5}$ (D) $\frac{-\sqrt{5}}{5}$

23. What are the first four positive solutions of the equation $\sin(2\theta) = \frac{1}{2}$?

(A) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$ (B) $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

(C) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}$ (D) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

24. Find all solutions in the interval $[0, 2\pi]$ for the equation $2\cos^2\theta - 1 = 0$.

(A) $\frac{\pi}{4}, \frac{7\pi}{4}$ (B) $\frac{3\pi}{4}, \frac{5\pi}{4}$ (C) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ (D) $\frac{\pi}{3}, \frac{5\pi}{3}$

25. A ship, off-shore from a vertical cliff known to be 200 feet high, takes a sighting of the top of a cliff. If the angle of elevation is found to be 15 degrees, approximately how far off-shore is the ship?

(A) 3000 feet (B) 1500 feet (C) 500 feet (D) 750 feet

26. Solve $log(x-1) + log(x+1) = 0$

a. $x = \sqrt{2}$
b. $x = -1, x = 1$
c. $x = 1$
d. $x = -\sqrt{2}, x = \sqrt{2}$
e. $x = 2$

27. Find the sum of all the zeros of the polynomial $f(x) = x^3 + 2x^2 - 5x - 6$

a. -5

b. -2

c. 0

d. 2

e. 6