

Lesson: How do you write the equation of a linear relationship given different types of information?

Eighth Grade Objective: **5.02 Write an equation of a linear relationship given: two points, the slope and one point on the line, or the slope and y-intercept.**

Lesson:

One of the main purposes of learning algebra is to be able to generalize, or create, equations and graphs to represent linear relationships. Using these graphs or equations, we can carry out patterns and display relationships more thoroughly and, believe it or not, more quickly, than we can by just using tables.

A linear relationship has a constant rate of change, which means it has a slope. We can use this slope to determine the equation of a line given one ordered pair.

If we are given the slope of a line is $\frac{1}{2}$ and the y-intercept is 6, we can use the form $y = ax + b$, where a is the slope and b is the y-intercept, to write the equation for the line. The slope of our line is $\frac{1}{2}$ so $a = \frac{1}{2}$, and the y-intercept is 6 so $b = 6$. Now we just substitute: $y = \frac{1}{2}x + 6$.

This means that you can plug in any value for x , solve the equation, and get a value for y . The x and y pair are an ordered pair on the graph of that equation.

Let's try:

Write the equation of a line using the given information:

1. slope -2, y-intercept 3

We know the slope and the y-intercept, so we will use the form $y=ax+b$ to write our equation.

$$a = -2, b = 3$$

$$\text{Now we substitute: } y = -2x + 3$$

2. slope = $\frac{1}{4}$, (0, -1)

We know the slope of the line and we have an ordered pair. Is that ordered pair the y-intercept? Yes, it is. We know because if we graphed that point, it would be on the y-axis. Another way that you can think about that is every time the x value is zero, you must be on the y-axis.

So, we will use the form $y=ax+b$ to write our equation.

$$a = \frac{1}{4}, b = -1$$

$$\text{Now we substitute: } y = \frac{1}{4}x - 1$$

Notice that we used a minus sign instead of an addition sign before the 1. If you are more comfortable writing $y = \frac{1}{4}x + (-1)$, that is also correct. Both let us know that the y-intercept is negative one, not positive one.

3. Cassidy is ordering flowers for Mother's Day. The store has a delivery fee of \$5.00 and each flower costs \$2.50. Write an equation to represent this situation and determine how many flowers she can buy her mother if she plans on spending \$20.00.

We know the slope, or rate of change, for the situation: for each flower, the cost increases by \$2.50, so the slope is +2.5.

Do we know the y-intercept? In the previous problem, we learned that when $x = 0$, the matching y-coordinate is the y-intercept, so what do x and y stand for? Because the x-value is our independent variable, we know that x is the number of flowers. The y-axis is our dependent variable and that is the cost (think about it this way, the number of flowers controls the cost). Since x is the number of flowers, what is the total cost even if she purchases no flowers? The delivery fee, \$5.00, is the y-intercept. Yes, it sounds silly, but in theory, the cost of the flowers begins at \$5.00, even if she doesn't buy anything. Since we know the slope and the y-intercept, we'll use the form $y = ax + b$ to write our equation. $a = 2.5$ and $b = 5$.

Now we substitute: $y = 2.5x + 5$

The question asked us one more thing – how many flowers could Cassidy buy if she plans on spending \$20.00? Since our total cost is our y-coordinate, we substitute \$20.00 in for y and solve:

$$20 = 2.5x + 5$$

Subtract 5 from each side:

$$15 = 2.5x$$

Divide both sides by 2.5:

$$6 = x$$

Cassidy can purchase 6 flowers.

What happens if we don't know the y-intercept? We can find it! Let's say we are given a slope of 3 and the ordered pair (4, 9). We begin the same way we do when we know the slope and y-intercept: $y = ax + b$. We know the slope, so $a = 3$, but we also know an x and a y-value. Looking at our ordered pair, the 4 is an x-value and the 9 is a y-value. So now we substitute:

$$y = ax + b$$

$$9 = 3(4) + b$$

And then simplify and solve:

$$9 = 12 + b$$

Subtract 12 from both sides:

$$-3 = b$$

Now we just re-write the equation using the slope ($a = 3$) and y-intercept ($b = -3$):

$$y = ax + b$$

$$y = 3x - 3 \text{ (or } y = 3x + (-3))$$

Let's try:

Write the equation of a line using the given information:

1. slope $\frac{1}{2}$, (-2, 5)

We know the slope, but we do not know the y-intercept.

We can use the equation $y = ax + b$ knowing that $a = \frac{1}{2}$, $x = -2$, $y = 5$.

$$\begin{aligned} & y = ax + b \\ \text{Substitute:} & 5 = \frac{1}{2}(-2) + b \\ \text{Simplify:} & 5 = -1 + b \\ \text{Solve by adding one to both sides:} & 6 = b \\ \text{Rewrite using a and b:} & y = \frac{1}{2}x + 6 \end{aligned}$$

2. slope -4, (1, -7)

We know the slope, but we do not know the y-intercept.

We can use the equation $y = ax + b$ knowing that $a = -4$, $x = 1$, $y = -7$.

$$\begin{aligned} & y = ax + b \\ \text{Substitute:} & -7 = -4(1) + b \\ \text{Simplify:} & -7 = -4 + b \\ \text{Solve by adding four to both sides:} & -3 = b \\ \text{Rewrite using a and b:} & y = -4x - 3 \end{aligned}$$

3. Marybeth is determining how much her empty backpack weighs. She knows each book weighs 12 pounds and that her bookbag weighs 42 pounds right now with three books inside. How much does her empty bookbag weigh?

The independent variable, x , is the number of books. The dependent variable, y , is the weight of her bookbag. We know that there are 3 books for a weight of 42 lbs, so we can use the ordered pair (3, 42).

We also know that the slope, or rate of change, is 12. For each book added to the bookbag, the weight of the bag increases by 12.

So, $a = 12$, $x = 3$, $y = 42$.

$$\begin{aligned} & y = ax + b \\ \text{Substitute:} & 42 = 12(3) + b \\ \text{Simplify:} & 42 = 36 + b \\ \text{Solve by subtracting 36} & \\ \text{from both sides:} & 6 = b \\ \text{Rewrite using a and b:} & y = 12x + 6 \end{aligned}$$

Marybeth's empty bookbag weighs six pounds.

What happens if we are just given two ordered pairs, (3, 0) and (6, 1)?

We can first find the slope by using the formula:

$$\begin{aligned} a &= (y_2 - y_1) / (x_2 - x_1) \\ a &= (1 - 0) / (6 - 3) \\ a &= 1/3 \end{aligned}$$

Now we can use the strategy we used with the last set of problems:

We know the slope, so $a = 1/3$, but we also know an x and a y -value. Looking at our ordered pair (we'll use (3, 0) for this example, but you can use whichever you'd like), the 3 is an x -value and the 0 is a y -value. So now we substitute:

$$\begin{aligned} & y = ax + b \\ 0 &= 1/3(3) + b \end{aligned}$$

And then simplify and solve: $0 = 1 + b$
Subtract 1 from both sides: $-1 = b$

Now we just re-write the equation using the slope ($a = 1/3$) and y-intercept ($b = -1$):
 $y = ax + b$
 $y = 1/3x - 1$ (or $y = 1/3x + (-1)$)

Let's try these:

Write the equation of a line using the given information:

1. (2, 9) and (-6, 1)

First find the slope: $a = (y_2 - y_1) / (x_2 - x_1)$
 $a = (1 - 9) / (-6 - 2)$
 $a = -8 / -8$
 $a = 1$

Now we know $a = 1$ and we'll use the ordered pair (2, 9) to find the equation*.

$y = ax + b$
Substitute: $9 = 1(2) + b$
Simplify: $9 = 2 + b$
Solve by subtracting 2
from both sides: $7 = b$
Rewrite using a and b: $y = 1x + 7$

*I chose (2,9) only because it looked easier, all positive numbers. You can choose whichever you'd like.

2. Renzo received a raise at work last week, but he doesn't remember by how much. He knows that he worked 3 hours on Tuesday, he made \$22.50 and he worked 5 hours on Wednesday and made \$37.50. How much would he make on Saturday if he is on the schedule to work 8 hours?

Our two variables are time and money. Time controls the amount of money made, so time is our x-variable and money is our y-variable. We can use the ordered pairs (3, 22.5) and (5, 37.5) to determine an equation Renzo can use to find how much money he will make on Saturday.

First we need to find the slope: $a = (y_2 - y_1) / (x_2 - x_1)$
 $a = (37.5 - 22.5) / (5 - 3)$
 $a = 15 / 2$
 $a = 7.5$

Now we know $a = 7.5$ and we'll use the ordered pair (3, 22.5) to find the equation.

$y = ax + b$
Substitute: $22.5 = 7.5(3) + b$
Simplify: $22.5 = 22.5 + b$
Solve by subtracting 2

from both sides: $0 = b$
 Rewrite using a and b: $y = 7.5x + 0$
 Substitute 8 in for x to find out how
 much money he will make Saturday: $y = 7.5(8) + 0$
 $y = 60$
 Renzo will make \$60 on Saturday.

Try these on your own:

Write the equation of a line using the given information:

- slope = -5, (0, 2)
- slope = $\frac{1}{4}$, (-4, 0)
- (-3, 7), (-1, 1)
- Jeff receives a salary plus commission for each ticket he sells to an upcoming hockey game. He knows he makes a total of \$175 if he sells 5 tickets and \$295 if he sells 13 tickets. Write an equation and determine:
 - How much Jeff makes if he sells zero tickets.
 - How many tickets Jeff would need to sell to make \$1000.

Check your answers:

1. We know the slope and the y-intercept, so we will use the form $y=ax+b$ to write our equation. $a = -5, b = 2$
 Now substitute: $y = -5x + 2$

2. We know the slope, but we do not know the y-intercept.
 We can use the equation $y = ax + b$ knowing that $a = \frac{1}{4}, x = -4, y = 0$.

Substitute: $y = ax + b$
 $0 = \frac{1}{4}(-4) + b$
 Simplify: $0 = -1 + b$
 Solve by adding one to both sides: $1 = b$
 Rewrite using a and b: $y = \frac{1}{4}x + 1$

3.
 First find the slope:
 $a = (y_2 - y_1) / (x_2 - x_1)$
 $a = (7 - 1) / (-3 - (-1))$
 $a = 6 / -2$
 $a = -3$

Now we know $a = -3$ and we'll use the ordered pair (-1, 1) to find the equation*.

Substitute: $y = ax + b$
 $1 = -3(-1) + b$

Simplify: $1 = 3 + b$
 Solve by subtracting 3
 from both sides: $-2 = b$
 Rewrite using a and b: $y = -3x - 2$

4. We can use the ordered pairs (5, 175) and (13, 295):
 First find the slope: $a = (y_2 - y_1) / (x_2 - x_1)$
 $a = (295 - 175) / (13 - 5)$
 $a = 120 / 8$
 $a = 15$

Now we know $a = 15$ and we'll use the ordered pair (5, 175) to find the equation*.

$y = ax + b$
 Substitute: $175 = 15(5) + b$
 Simplify: $175 = 75 + b$
 Solve by subtracting 75
 from both sides: $100 = b$
 Rewrite using a and b: $y = 15x + 100$

a: \$100 - this is the y-intercept,
 b: substitute 1000 for y: $1000 = 15x + 100$
 solve by subtracting 100 from
 both sides: $900 = 15x$
 divide both sides by 15: $60 = x$

Jeff would need to sell 60 tickets to make \$1000.

Quiz Yourself:

Write the equation of a line using the given information:

1. slope = -1, (0, 4)
2. slope = 2, (6, 10)
3. (-8, 4), (-6, 8)
4. Delanie washes two cars in one hour and seven cars in 3 ½ hours. How many cars can she wash in an 8-hour day?

Check your answers:

1. $y = -1x + 4$ or $y = -x + 4$
2. $y = 2x - 2$
3. $y = 2x + 20$

4. $y = \frac{1}{2}x + 0$, Delanie can wash 16 cars in one eight-hour day.