

Enrichment Lesson: How do you write the equation of a linear relationship given different types of information?

Eighth Grade Objective: **5.02 Write an equation of a linear relationship given: two points, the slope and one point on the line, or the slope and y-intercept.**

Lesson:

One of the main purposes of learning algebra is to be able to generalize, or create, equations and graphs to represent linear relationships. Using these graphs or equations, we can carry out patterns and display relationships more thoroughly and, believe it or not, more quickly, than we can by just using tables.

We can create equations for linear data, as long as we have two points on the given line.

There are three ways we will focus on to write the equation of a linear relationship:

Slope-intercept form:  $y = ax + b$ ;  $a$  is your slope,  $b$  is your y-intercept

Point-slope form:  $y - y_1 = a(x - x_1)$ ;  $a$  is your slope  $(x_1, y_1)$  are ordered pairs

Standard form:  $Ax + By = C$ ; where  $A$ ,  $B$  and  $C$  are integer coefficients

Let's try using each of the forms:

1. Gabe is driving 65 miles per hour to visit his cousin in New York. The trip is a total of 550 miles. Write an equation Gabe can use to determine his distance from his cousins house at any given time.

What do we know:

a. Gabe is traveling at a rate of 65 mph, which is our slope.

b. Our independent variable ( $x$ ) is time, or dependent variable ( $y$ ) is distance.

c. At time zero, Gabe was 550 miles from his home  $(0, 0)$ , so the y-intercept must be 0.

If we know the slope and the y-intercept, the easiest form to use is slope-intercept:  $y = ax + b$ . Substitute:  $y = 65x + 0$  or just  $y = 65x$ .

2. Dierdra wants to make \$100 selling hot beverages at the school football game. She is charging \$0.50 per cup of coffee and \$0.75 per cup of hot chocolate. Write an equation that can be used to determine how many cups of each she must sell to reach her goal.

What do we know:

a. A cup of coffee is \$0.50.

b. A cup of hot chocolate is \$0.75.

c. She wants to make \$100.

We do not know a slope, and either could be written as the independent variable.

We can define  $x$  as the number of cups of coffee and  $y$  as the number of cups of hot chocolate. This means that if we multiply the number of cups of each sold by their respective prices, adding the totals, we want to arrive at a total of \$100.

So,  $0.50x + 0.75y = 100$ .

3. Karissa is making bracelets to sell at a craft fair. The fair charges a certain amount to have a booth. She knows that to make \$25, she has to sell 20 bracelets and to make \$40 she has to sell 24 bracelets. How much does the fair charge for the booth?

What do we know:

- a. Our independent variable (x) is the number of bracelets sold.
- b. Our dependent variable (y) is amount of money made at the fair.
- c. We have two ordered pairs (20, 25) and (24, 40).

We can find the slope given that information:

$$a = (y_2 - y_1) / (x_2 - x_1)$$

$$a = (25 - 40) / (20 - 24)$$

$$a = -15 / -4$$

$$a = 3.75, \text{ the cost per bracelet is } 3.75.$$

We can use the slope and one of the ordered pairs to write an equation in point slope form:

$$y - y_1 = a(x - x_1)$$

$$y - 25 = 3.75(x - 20)$$

It is possible to transform each equation to look however you are most comfortable. For example, if you prefer slope-intercept form to point-slope form, you can take the above equation, distribute the 3.75 to x and -20, then add 25 and arrive at  $y = 3.75x - 50$ . This allows us to clearly see that the y-intercept is -50, therefore, if Karissa sells zero bracelets, she is still charged \$50 for the booth rental at the fair.

Try these on your own:

1. Joshua is planning a cook-out for his brother's birthday party. Hamburger costs \$4 per pound and each pound makes 3 burgers. Hot dogs cost \$3 per pound. Write an equation to determine how many pounds of hamburger and hot dogs could he buy if he must spend exactly \$30?

2. Companies must take into account their operating costs when determining profit. If plants unlimited sells 10 plants per day, their profit is \$100. If they sell 20 plants per day, their profit is \$250. How much does each plant cost and how many plants would need to be sold to make a profit of \$1000?

3. A voice-over company charges \$1500 per referral plus \$250 per hour their clients work. How many hours does a client work who makes \$3500?

Check your answers:

1. What do we know?
  - a. hamburger costs \$4 per pound
  - b. hot dogs cost \$3 per pound
  - c. Joshua has \$30.

d. The cost of one affects the other, but either could be the independent variable, making this most like example 2.

We can define  $x$  as the number of pounds of hamburger and  $y$  as the number of pounds of hot dogs. When we multiply each by their respective costs and add, we should get \$30.

$$Ax + By = C$$

$$4x + 3y = 30$$

2. What do we know?

a. Number of plants sold is our independent variable ( $x$ ), profit is our dependent variable ( $y$ ).

b. (10, 100) and (20, 250) are two ordered pairs we can use.

We can find the slope given that information:

$$a = (y_2 - y_1) / (x_2 - x_1)$$

$$a = (250 - 100) / (20 - 10)$$

$$a = 150 / 10$$

$$a = 15, \text{ the cost per plant is } \$15.$$

We can use the slope and one of the ordered pairs to write an equation in point slope form:

$$y - y_1 = a(x - x_1)$$

$$y - 250 = 15(x - 20)$$

$$y - 250 = 15x - 300$$

$$y = 15x - 50$$

Substitute \$1000 in for  $y$ :

$$1000 = 15x - 50$$

$$1050 = 15x$$

$$70 = x, \text{ 70 plants would have to be sold.}$$

3. What do we know?

a. A flat fee of \$1500 is charged. If the client works 0 hours, the company makes \$1500.

b. A fee of \$250 per hour is charged. This is the slope.

We can use slope-intercept

form to write an equation:

$$y = ax + b$$

$$y = 250x + 1500$$

We can substitute 3500 for  $y$  to determine the number of hours worked:

$$3500 = 250x + 1500$$

Subtract 1500 from both sides:  $2000 = 250x$

Divide by 250:

$$8 = x, \text{ the client would need to work 8 hours.}$$

Quiz Yourself:

1. If a publishing company sells 1000 books, they make a profit of \$10,000. If the company sells 500 books, they make a profit of \$2000. Write an equation that can be used to determine the profit for any number of books sold.

2. Low-grade gasoline costs \$3.00 per gallon and high-grade gasoline costs \$3.20 per gallon. Write an equation to determine how much of each type of gasoline can a business owner purchase if the business owner must spend exactly \$5000.
3. A family takes \$100 per month out of a bank account to save for future college expenses. The account has \$4000 in it to begin with. Write an equation that can be used to determine how much money will be in the account after 1 year.

Check your answers:

1. Slope-intercept form:  $y = 16x - 6000$   
 Standard form:  $-16x + y = -6000$  or  $16x - y = 6000$   
 Point-slope form:  $y - 2000 = 16(x - 500)$  or  $y - 10,000 = 16(x - 1000)$
2.  $x =$  low-grade gasoline,  $y =$  high-grade gasoline  
 Standard form:  $3x + 3.2y = 5000$   
 Slope-intercept form:  $y = -15/16x + 1562.5$
3.  $y = -100x + 4000$  or  $y = 4000 - 100x$   
 The account will have \$2800 in it after one year, assuming no other withdrawals or deposits were made.