

Lesson: Line of best fit.

Eighth Grade Objective: 4.02 Approximate a line of best fit for a given scatterplot; explain the meaning of the line as it relates to the problem and make predictions.

Lesson:

In Goal 4.01, we learned how to graph scatterplots using appropriate scale and interval. We also practiced recognizing independent and dependent variables. In this lesson, we are going to superimpose a line onto our scatterplots that we can use to make predictions.

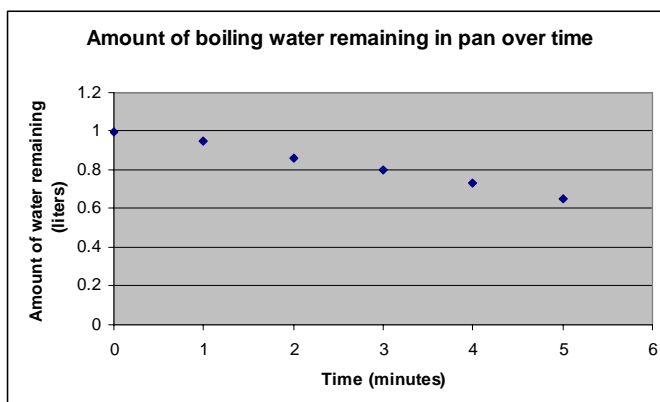
Using a line to represent data that is almost linear is called using a line of best fit. We are using a line to represent the trend of the data, not to represent each data point exactly. When we fit a line to our data it is important to keep a few things in mind. The first, lines are straight. There is no such thing as a crooked line. The second, we are trying to use a line to represent the trend in our data, that means that about half of our points should be above the line and about half of our points should be below the line. It is ok if some points are on the line, and it is ok if there are no points on the line.

Let's start:

We start by graphing the data:

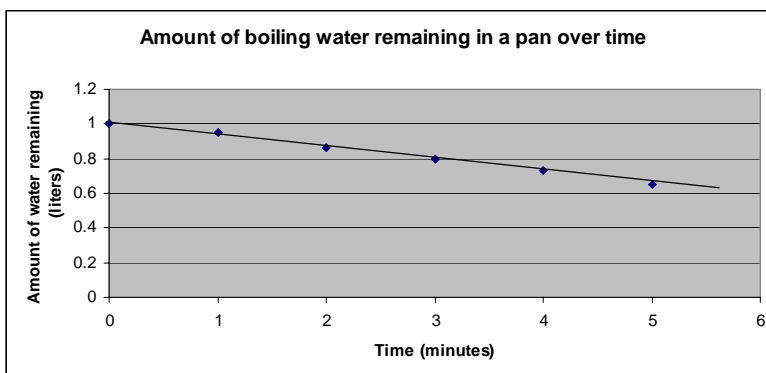
Time (minutes)	Amount of boiling water remaining in pan (liters)
0	1
1	0.95
2	0.86
3	0.8
4	0.73
5	0.65

The time is the independent variable, the amount of water remaining is the dependent variable. Let's use a scale of 0-6 on the x-axis and 0-1.2 on the y-axis. We'll count by one on the x-axis and by 0.2 on the y-axis. It is ok if you chose a slightly different scale or interval.



Now, if you have one, place a piece of uncooked spaghetti on your graph to represent your line. Spaghetti is great to use because it doesn't bend much! Once you have about ½ of your points above the line and ½ below, tape your spaghetti to the paper.

If you don't have spaghetti handy, use a straightedge to place a line on your graph that has about ½ the data points above the line and ½ below the line. An edge of a sheet of paper is a great straightedge tool.



Your line could be in a slightly different place, just keep in mind that your line is supposed to represent your data. The line should have a y-intercept of approximately 1 and a distinct negative slope.

Once you are able to draw a line of best fit, we need to be able to write an equation for the line. The most common way of doing this is to find the slope using two points on the line. In the graph above, we can use (0,1) and (3, 0.81).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{1 - 0.81}{0 - 3}$$

$$m = 0.19/-3 \text{ or about } -0.06$$

Now, we can use slope-intercept form of an equation to find the y-intercept. We need one ordered pair and the slope. Let's use (0,1) and $m = -0.06$.

$$y = mx + b$$

$$1 = -0.06(0) + b$$

$$1 = 0 + b$$

$$1 = b$$

In this case, we actually used our y-intercept as our point. That will not always be the case, but the procedure will remain the same. Now we rewrite the equation with our m and b values:

$$y = -0.06x + 1$$

We can use this equation to make predictions. Looking back at the graph, we can recall that our x variable stands for time in minutes and our y variable stands for amount of water remaining in liters. So how many liters will remain after 10 minutes?

$$\text{Time} = x, \text{ so } x = 10$$

$$y = -0.06x + 1$$

$$y = -0.06(10) + 1$$

$$y = 0.4 \text{ liters remaining}$$

How much time will it take to have only 0.2 liters remaining?

$$\text{Liters} = y, \text{ so } y = 0.2$$

$$y = -0.06x + 1$$

$$0.2 = -0.06x + 1$$

$$-0.8 = -0.06x$$

$$x = 13.3 \text{ minutes}$$

Try these on your own!

Graph the data and draw a line of best fit.

1.

Time (hours)	Distance Walked (miles)
0	0
1	3
2	7
3	10
4	14

a. Graph the data and draw a line of best fit.

b. Write an equation for your line of best fit.

c. Predict how much time it takes to walk 20 miles.

d. Predict how far the person could walk if they walked for a full day.

2.

Miles Driven Gallons of Gas Left in

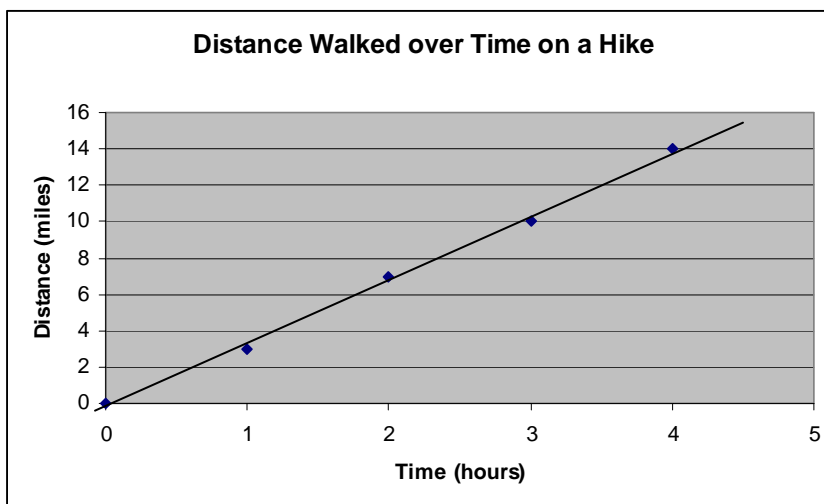
	Tank
0	15
25	14
53	13
78	12
105	11
136	10

- Graph the data and draw a line of best fit.
- Write an equation for your line of best fit.
- Predict how many miles have been driven if there are 5 gallons of gas left in the tank.
- Predict how many gallons of gas are left in the tank after driving 200 miles.

Check your answers:

Remember, scales and intervals may vary. Your scale is ok if all the data points fit on the graph and the plotted points take up most of the width and the height of the graph (there's not a lot of unused space). Your interval is probably ok as long as you count by the same thing on each axis. Also keep in mind that the placement of the line can vary slightly, your slope and y-intercept may be a little bit larger or smaller than the one shown in the key, making your answers for C and D slightly different. The answer, though, should be close.

1. a.



b. Let's use ordered pairs (0,0) and (2,7) to find our slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{7 - 0}{2 - 0}$$

$$m = 7/2 \text{ or } 3.5$$

Let's use (0,0) to find our y-intercept (or, if you will notice, 0 must be our y-intercept, since our line crosses the y-axis at zero, but we'll show the math anyway for practice!). We, of course, need to use the slope we just determined, 3.5 or $7/2$.

$$y = mx + b$$

$$0 = 3.5(0) + b$$

$$0 = 0 + b$$

$$0 = b$$

Now we rewrite the equation with our m and b values:

$$y = 3.5x + 0$$

c. Looking back at the graph, we recall that distance is the y variable and time is the x variable. If distance = 20 miles, then $y = 20$. We can substitute this into our equation:

$$y = 3.5x + 0$$

$$20 = 3.5x + 0$$

$$x = 5.7 \text{ hours}$$

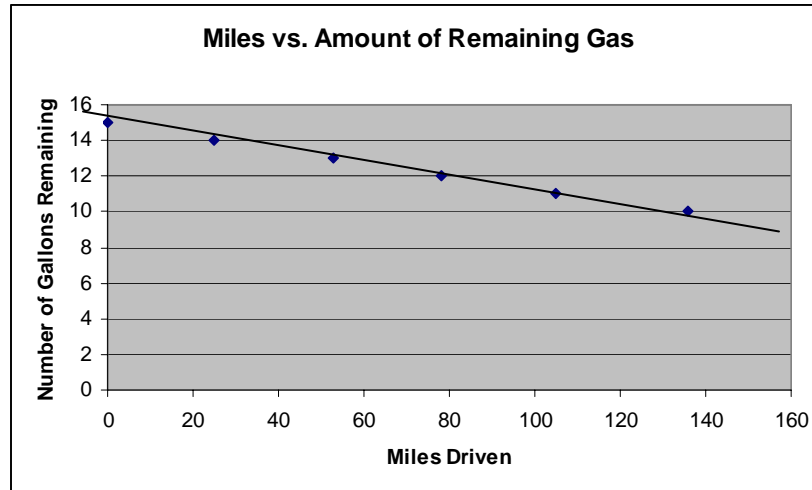
d. Recall that time in hours is the x variable. If time is 24 hours, then $x = 24$. We can substitute this into our equation:

$$y = 3.5x + 0$$

$$y = 3.5 (24) + 0$$

$$y = 84 \text{ miles}$$

2. a.



b. Let's use ordered pairs (78, 12) and (53, 13) to find our slope (the line appears to go through those two points and we go back to the chart to see what they are exactly):

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{13 - 12}{53 - 78}$$

$$m = 1/-25 \text{ or } -1/25 \text{ or } -0.04$$

Let's use (78, 12) to find our y-intercept. We, of course, need to use the slope we just determined, -0.04.

$$y = mx + b$$

$$12 = -0.04(78) + b$$

$$12 = -3.12 + b$$

$$15.12 = b$$

Now we rewrite the equation with our m and b values:

$$y = -0.04x + 15.12$$

c. Recall, by looking at the graph, that the x variable is the number of miles driven and the y variable is the amount of gas left in the car. If there are 5 gallons of gas left in the tank, $y = 5$. We can use our equation to find x:

$$y = -0.04x + 15.12$$

$$5 = -0.04x + 15.12$$

$$-10.12 = -0.04x$$

$$253 = x$$

The car will have gone approximately 253 miles when there is 5 gallons remaining in the tank.

d. If we want to know how many gallons will be left after driving 200 miles, we substitute 200 in for x:

$$y = -0.04x + 15.12$$

$$y = -0.04(200) + 15.12$$

$$y = 7.12 \text{ gallons}$$

There is approximately 7.12 gallons of gas left in the car after driving 200 miles.

Quiz yourself!

1.

Day	Amount of Money in Checking Account
0	300
5	280
10	240
15	190
20	130
25	100
30	70

- Graph the data and draw a line of best fit.
- Write an equation for your line of best fit.
- Predict how much money will be left in the account after 27 days.
- Predict how many days it will take for the account to reach zero

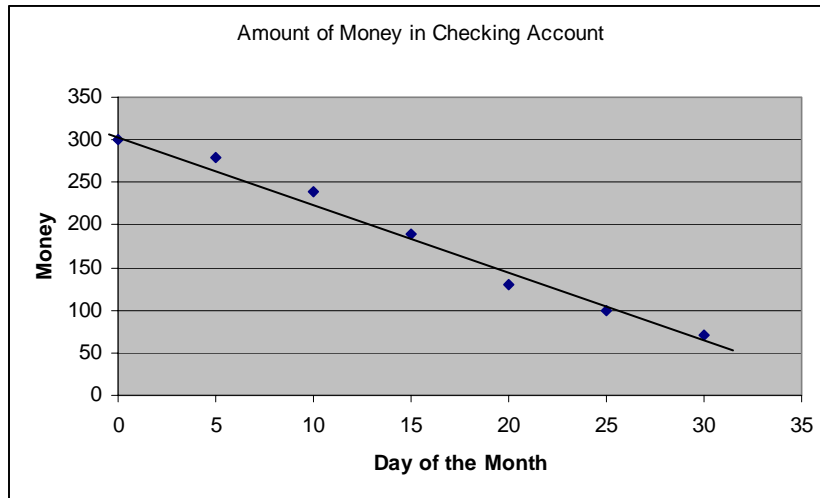
2.

Annual Income	Years of Education
\$14,000	8
\$16,000	9
\$22,000	12
\$28,000	14
\$35,000	16
\$47,000	18

- Graph the data and draw a line of best fit.
- Write an equation for your line of best fit.
- Predict how many years of education it will take to reach an annual income of \$60,000.
- Predict how the annual income of a person with 15 years of education.

Check your answers:

1. a.



b. Let's use ordered pairs (0, 300) and (25, 100) to find our slope (the line appears to go through those two points and we go back to the chart to see what they are exactly):

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{300 - 100}{0 - 25}$$

$$m = 200/-25 \text{ or } -8$$

Let's use (0, 300) to find our y-intercept. We, of course, need to use the slope we just determined, -8. You may also notice that we actually already have the y-intercept, we'll calculate just for practice.

$$y = mx + b$$

$$300 = -8(0) + b$$

$$300 = 0 + b$$

$$300 = b$$

Now we rewrite the equation with our m and b values:

$$y = -8x + 300$$

c. Recall, by looking at the graph, that the x variable stands for day of the month and y stands for amount of money. If we are finding the amount of money in the account after 27 days, $x = 27$ and we substitute to find y:

$$y = -8x + 300$$

$$y = -8(27) + 300$$

$y = 84$, there is about \$84 left in the account after 27 days.

d. We are interested in how many days it will take for the account to reach zero, therefore $y = 0$.

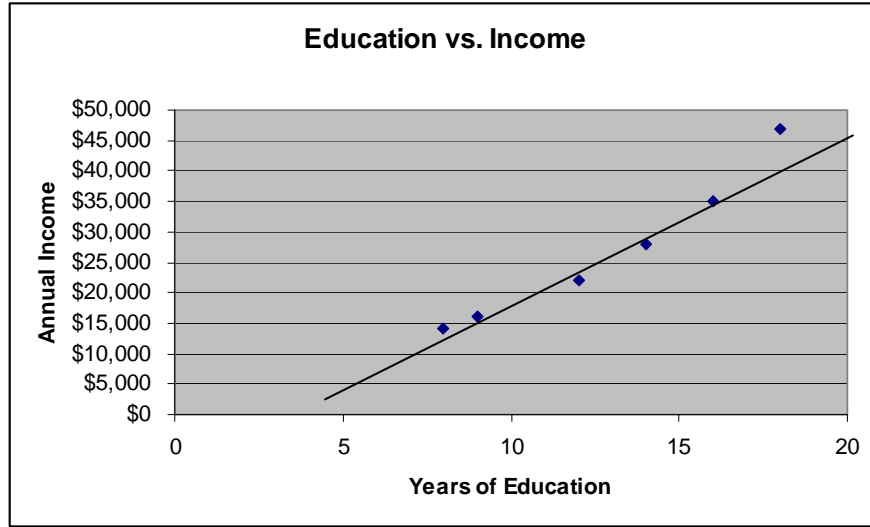
$$y = -8x + 300$$

$$0 = -8x + 300$$

$$-300 = -8x$$

$x = 37.5$ days. After 37.5 days, the account will reach zero

2. a.



b. Let's use ordered pairs (9, 16,000) and (16, 35,000) to find our slope (the line appears to go very close to those two points and we go back to the chart to see what they are exactly):

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{35,000 - 16,000}{16 - 9}$$

$$m = 19,000/7, \text{ approximately } 2714$$

Let's use (9, 16,000) to find our y-intercept. We, of course, need to use the slope we just determined, 2714. You may also notice that we actually already have the y-intercept, we'll calculate just for practice.

$$y = mx + b$$

$$16,000 = 2714(9) + b$$

$$16,000 = 24,426 + b$$

$$-8426 = b$$

Now we rewrite the equation with our m and b values:

$$y = 2714x + (-8426)$$

c. Recall, by looking at the graph, that income is the y-variable and years of education is the x variable. If we are interested in knowing how many years of education it takes to earn \$60,000, we plug in \$60,000 for y and solve for x.

$$y = 2714x + (-8426)$$

$$60,000 = 2714x + (-8426)$$

$$68,426 = 2714x$$

x is approximately 25. It would theoretically take 25 years of education to have an income of \$60,000.

d. If we are going to predict the annual income of a person with 15 years of education, we substitute 15 for x and solve for y.

$$y = 2714 (15) + (-8426)$$

$y = \$32,384$, a person with 15 years of education can expect to make \$32,384 based on this model.