Lesson: Dilations

Eighth Grade Objective: 3.03 Identify, predict, and describe dilations in the coordinate plane.

Lesson:
Dilations are simply a size change by a scale factor.

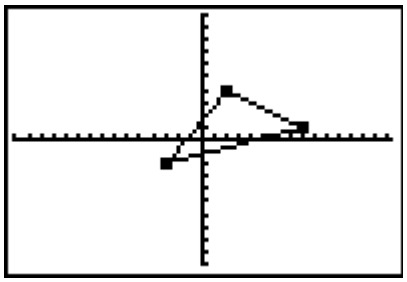
In the business world, documents can be reduced or enlarged by a scale factor on a copying machine – both sides of your math book can be reduced to copy on one 8 ½ x 11 sheet of paper, or a CD cover could be enlarged so that when copied, it takes up most of a sheet of paper.

In the medical world, eye doctors instill drops in people's eyes to dilate their pupils so the doctor will have an easier time looking at the structures inside someone's eye (the pupil is the dark, center part of the eye). In this case, eye doctors want their patient's pupils to become larger. Pupils can also constrict (become smaller) if a person were to be in an environment with very bright lights.

In the mathematics world, graphs (and other pictures, models, etc) can be dilated. The pre-image (first image) will look exactly like the image (second image), just a different size. The pre-image can either be reduced or enlarged, but each dimension must be changed by the same scale factor.

Let's try:
Did a dilation occur?

1.



Pre-image
(2, 4), (8, 1), (-3, -2)

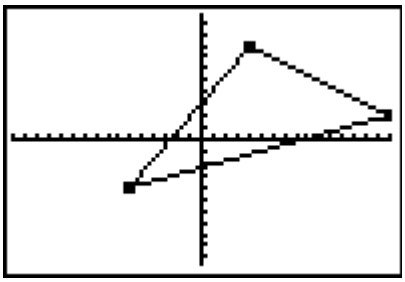
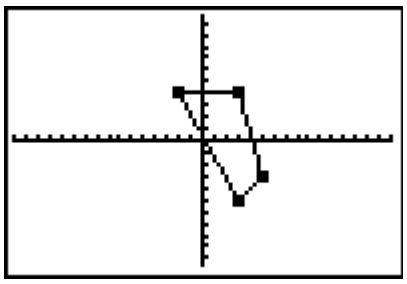


Image
(4, 8), (16, 2), (-6, -4)

Yes, a dilation occurred. The image is just a larger version of the pre-image. The figures are still similar (same angles, proportional sides). Each ordered pair in the pre-image was multiplied by 2.

2.



Pre-image
(3, 4), (5, -3), (3, -5), (-2, 4)

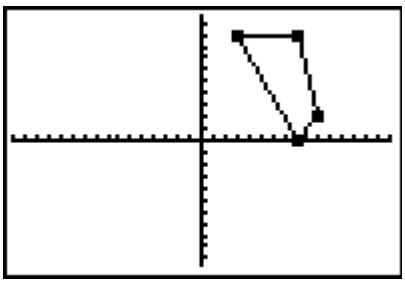
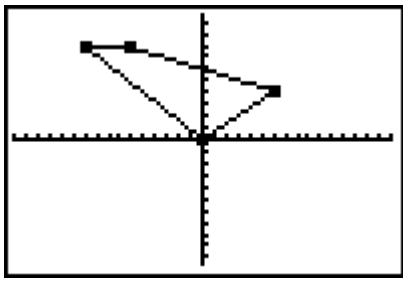


Image
(8, 9), (10, 2), (8, 0), (3,9)

No, a dilation did not occur, a translation (slide) occurred. Each coordinate was increased by five (+5), the size of the figure did not change.

3.



Pre-image
(6, 4), (-6, 8), (-10, 8), (0, 0)

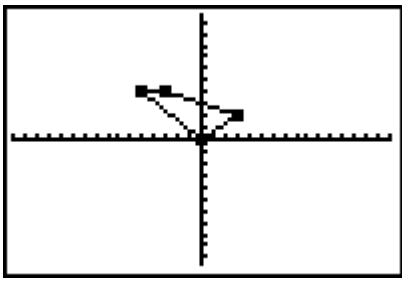


Image
(3, 2),(-3, 4), (-5, 4), (0, 0)

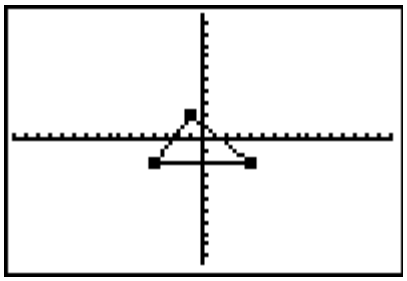
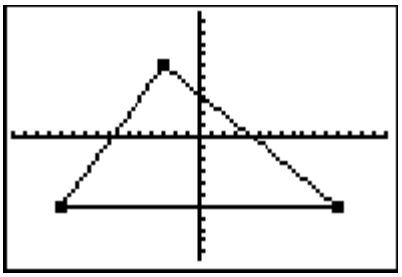
Yes, a dilation occurred. The image is just a smaller version of the pre-image. The figures are still similar (same angles, proportional sides). Each coordinate was multiplied by ½ .

To perform a dilation, you would start with the ordered pairs from the pre-image and multiply each coordinate by the scale factor.

Let's try:

1. Dilate the triangle by a scale factor of 3: (4, -2), (-4, -2), (-1, 2).

Multiply each coordinate by the scale factor (3): (12, -6), (-12, -6), (-3, 6)
Graph to check to make sure your figures are similar:

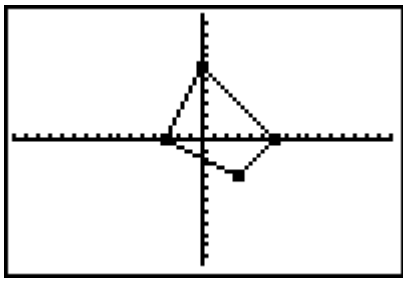
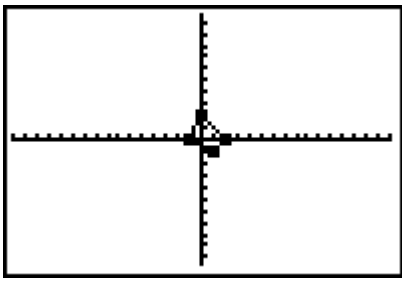



Pre-image Image

Yes, the figures are similar (same angles, proportional sides) and we have performed a dilation!

2. Dilate the quadrilateral by a scale factor of 1/3: (-3, 0), (0, -3), (6, 0), (0, 6)

Multiply each coordinate by the scale factor (1/3): (-1, 0), (0, -1), (2, 0), (0, 2)
Graph to check to make sure the figures are similar:

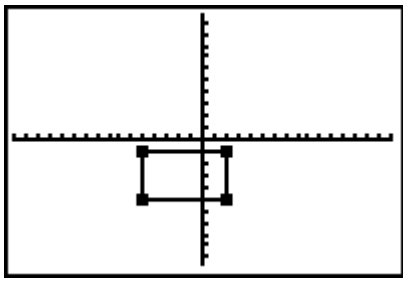
Pre-image Image

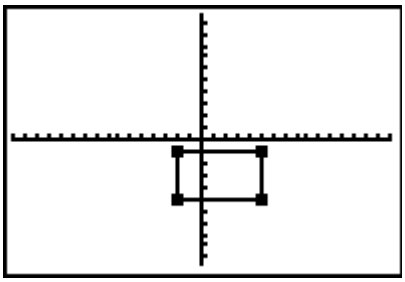
Yes, the figures are similar (same angles, proportional sides) and we have performed a dilation!

Try these on your own:

Tell whether or not the following are dilations. If they are dilations, what is the scale factor? If they are not dilations, what type of transformation took place?

1.

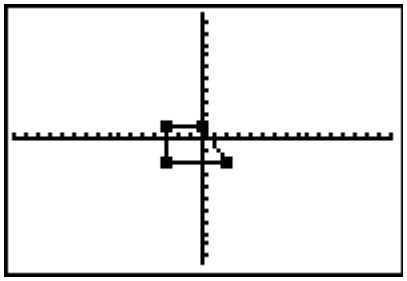


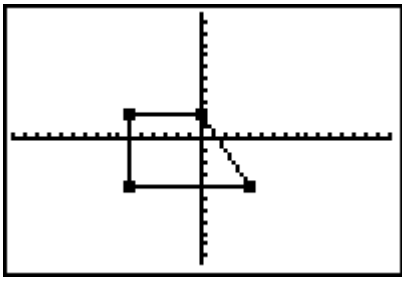


Pre-Image
(-5, -1), (2, -1), (2, -5), (-5, -5)

Image
(5, -1), (-2, -1), (-2, -5), (5, -5)

2.





Pre-image
(-3, 1), (0, 1), (2, -2), (-3, -2)

Image
(-6, 2), (0, 2), (4, -4), (-6, -4)

3. Dilate the triangle with vertices (5, 4), (3, -2), (10, -2) by a scale factor of ½ .

4. Dilate the parallelogram with vertices (-1, 3), (2, 3), (-1, -4), (-4, -4) by a scale factor of four.

5. Give an example of a scale factor that would produce an image larger than the pre-image.

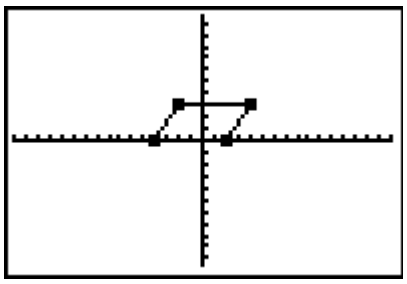
6. Give an example of a scale factor that would produce an image smaller than the pre-image.

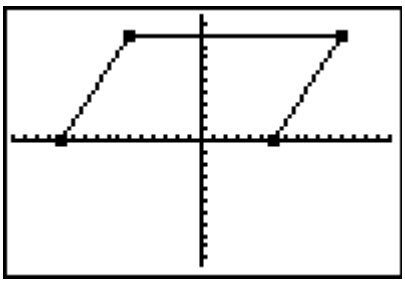
Check your answers:

1. This is not a dilation. This is a reflection over the y-axis (notice all the x-values have different signs).
2. This is a dilation by a scale factor of 2. Each coordinate has been multiplied by 2.
3. The new triangle would have vertices: (2.5, 2), (1.5, -1), (5, -1).
4. The new parallelogram would have vertices: (-4, 12), (8, 12), (-4, -16), (-16, -16).
5. Any scale factor greater than 1 will produce an image larger than the pre-image.
6. Any scale factor greater than zero but less than one will produce an image smaller than the pre-image.

Quiz Yourself!

1. Is the image a dilation of the pre-image? If yes, what is the scale factor?

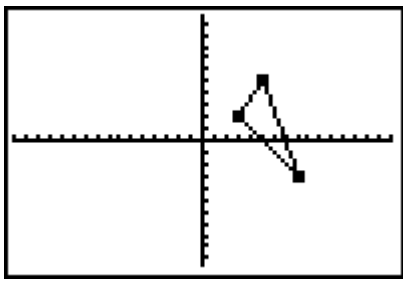


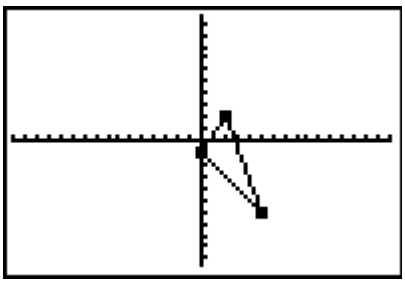


Pre-image
(-2, 3), (4, 3), (2, 0), (-4, 0)

Image
(-6, 9), (12, 9), (6, 0), (-12, 0)

2. Is the image a dilation of the pre-image? If yes, what is the scale factor?





Pre-image
(8, -3), (3, 2), (5, 5)

Image
(5, -6), 0, -1), (2, 2)

3. Dilate the triangle with vertices (-4, 2), (4, -2) and (-4, -2) by a scale factor of ½.

4. Dilate the quadrilateral with vertices (2, 4), (7, 4) (5, 0) and (0, 0) by a scale factor of 2.

5. Challenge! What is the area of a triangle whose pre-image has vertices (1, 1), (3, 1) and (3, 4) and is dilated by a scale factor of 3?

Check your answers:

1. Yes, the image is a dilation of the pre-image and the scale factor is 3.
2. No, the image is not a dilation, it is a translation, the x- and y-values were both changed by subtracting 3.
3. The image would have vertices (-2, 1), (2, -1), (-2, -1).
4. The image would have vertices (4, 8), (14, 8), (10, 0) and (0, 0).
5. The area is 27 units. The new vertices are (3, 3), (9, 3) and (9, 12). Graphing those ordered pairs, you can count the base length of 6, height 9. a = ½ bh, which is 27 units.