Lesson: Dilations

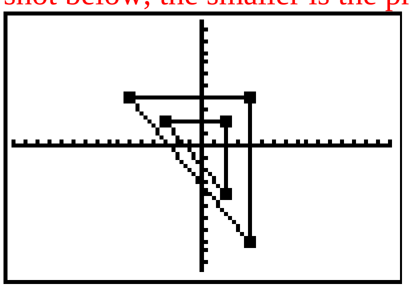
Eighth Grade Objective: 3.03 Identify, predict, and describe dilations in the coordinate plane.

Lesson:
Dilations are simply a size change by a scale factor. Using ordered pairs on a graph, you can increase or decrease a figure by multiplying each coordinate by any given factor. The resulting figure (image) is similar to the original image (pre-image).

Taking this one step further, we can use dilations to determine the effect of a size change on the area of two dimensional figures, determine geometric probabilities and apply dilations when performing multiple transformations.

1. By what factor does the area change when a triangle with vertices (2, 2), (2, -4), (-3, 2) is dilated by a scale factor of two?

The vertices of the image are (4, 4), (4, -8), (-6, 4). This is the larger figure in the screen shot below, the smaller is the pre-image.



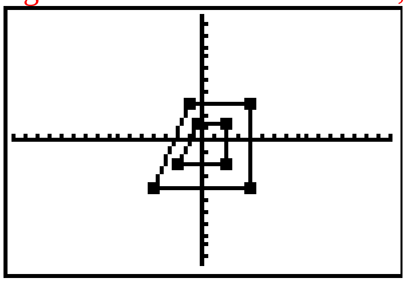
The base of the image is the distance from (4, 4) to (-6, 4), 10 units.
The height of the image is the distance from (4, 4) to (4, -8), 12 units.
Using the formula a = ½ bh, the area of the larger triangle is 60 square units.

The base of the pre-image is the distance from (2, 2) to (-3, 2), 5 units.
The height of the pre-image is the distance from (2, 2) to (2, -4), 6 units.
Using the formula a = ½ bh, the area of the smaller triangle is 15 square units.

The ratio of the area of the pre-image to the area of the image is 15:60 or 1:4. The area increased by a factor of four.

2. By what factor does the area change when a trapezoid with vertices (-1, 3), (4, 3), (4, -4), (-4, -4) is dilated by a scale factor of ½ ?

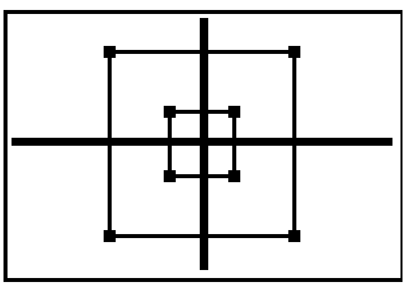
The vertices of the image are (-0.5, 1.5), (2, 1.5), (2, -2), (-2, -2). This is the smaller figure in the screen shot below, the larger is the pre-image.



The bases of the image are the distances from (2, -2) to (-2, -2) and (-0.5, 1.5) to (2, 1.5), 4 and 2.5 units, respectively. The height is the distance from (2, 1.5) to (2, -2), 3.5 units. Using the formula $a = \frac{1}{2} h (b_1 + b_2)$, the area of the smaller trapezoid, the image, is 11.375 square units.
The bases of the pre-image are the distances from (4, -4) to (-4, -4) and (-1, 3) to (4, 3), 8 and 5 units, respectively. The height is the distance from (4, 3) to (4, -4), 7 units. Using the formula $a = \frac{1}{2} h (b_1 + b_2)$, the area of the larger trapezoid, the pre-image, is 45.5 square units.
The ratio of the area of the pre-image to the area of the image is 45.5:11.375 or 4:1, the area decreased by a scale factor of ¼ .

3. A square having vertices (30, 30), (30, -30), (-30, -30), (-30, 30) is drawn onto a large sheet of graph paper. The figure is dilated by a factor of 1/3. If a penny is dropped onto the original graph, what is the probability the center of the penny will land inside the pre-image, but outside the image?

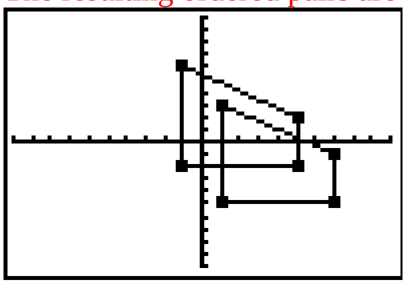
The vertices of the image are (10, 10), (10, -10), (-10, -10), (-10, 10), the image is the smaller figure in the screen shot and the pre-image is the larger.



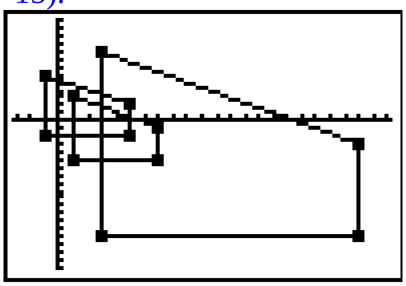
The goal is for the center of the penny to land in the space between the two figures, or the area created when the area of the smaller square is removed from the area of the larger square.
The area of the larger square is 60 x 60 = 3600 square units.
The area of the smaller square is 20 x 20 = 400 square units.
The area of the space in between is 3600 – 400 = 3200.
The probability of the penny landing in that space is 3200/3600 = 8/9 or 0.8889 or approximately 88.9%

4. What are the vertices of the image of a trapezoid located at (-1, 6), (5, 2), (5, -2), (-1, -2) after it has been translated two units right and three units down and dilated by a scale factor of 3?

When translating, the image remains congruent to the pre-image. The pre-image is moved a determined number of units – in this case two right and three down. Another way of saying this is +2 in the x direction and -3 in the y direction.
The resulting ordered pairs are (1, 3), (7, -1), (7, -5), (1, -5).

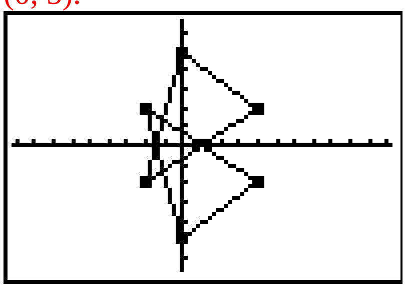


Those coordinates are then dilated by a scale factor of three: (3, 9), (21, -3), (21, -15), (3, -15).

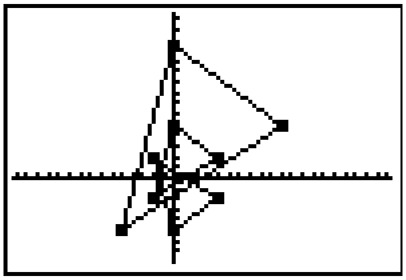


5. What are the vertices of the image of a triangle located at (-2, 2), (4, -2), (0, -5) after it has been reflected across the x-axis and dilated by a scale factor of 2.5?

When reflecting, the image remains congruent to the pre-image, oriented differently. When reflecting over the x-axis, the x-coordinates of the ordered pairs remain the same and the y-coordinates are multiplied by -1. The resulting ordered pairs are (-2, -2), (4, 2), (0, 5).



Those coordinates are then dilated by a scale factor of 2.5: (-5, -5), (10, 5), (0, 12.5).



Let's try:

1. By what factor does the area change when a triangle with vertices (-2, 4), (4, 4), (-2, 7) is dilated by a scale factor of 3?

2. By what factor does the area change when a parallelogram with vertices (0, -4), (1, -2), (5, -2), (4, -4) is dilated by a scale factor of ¼ ?

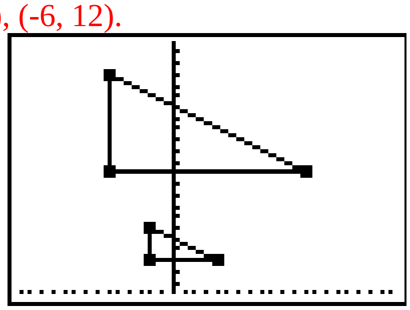
3. A triangle with vertices (-1, 4), (-1, -1), (3, -1) is dilated by a scale factor of 2. What is the probability a dart thrown at the larger triangle lands within the smaller triangle?

4. A rectangle with vertices (-1, 0), (-4, 0), (-4, 7), (-1, 7) is dilated by a scale factor of ½ and reflected over the y-axis. What are the vertices of the image?

5. A triangle with vertices (-2, 0), (-5, 1), (-3, 3) is translated one unit left and two units down and dilated by a scale factor of 2. What are the vertices of the image?

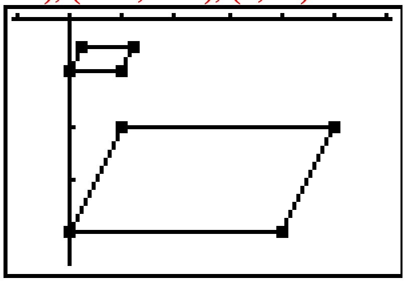
Check your work:

1. In the screen shot, the image is the larger triangle, with vertices (-6, 12), (12, 12), (-6, 21), (-6, 12).



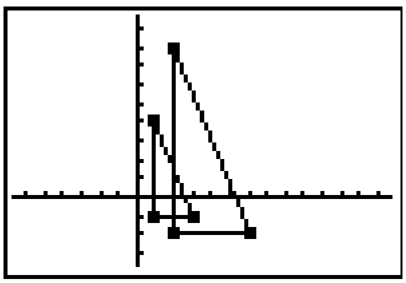
The base of the pre-image is the distance from (-2, 4) to (4, 4), 6 units.
The height of the pre-image is the distance from (-2, 4) to (-2, 7), 3 units.
Area of a triangle = ½ bh, or 9 square units.
The base of the image is the distance from (-6, 12) to (12, 12), 18 units.
The height of the image is the distance from (-6, 12) to (-6, 21), 9 units.
Area of a triangle = ½ bh, or 81 square units.
The ratio of the area of the pre-image to the area of the image is 9:81, 1:9. The area increased by a factor of 9.

2. In the screen shot, the image is the smaller parallelogram with vertices (0, -1), (0.25, -0.5), (1.25, -0.5), (1, -1).



The base of the pre-image is the distance from (0, -4) to (4, -4), 4 units.
The height of the pre-image is the perpendicular distance from one base to the other, 2 units.
Area of a parallelogram = bh, or 8 square units.
The base of the image is the distance from (0, -1) to (1, -1), 1 unit.
The height of the image is the perpendicular distance from one base to the other, ½ .
Area of a parallelogram = bh, or ½ square unit.
The ratio of the area of the pre-image to the area of the image is 8: ½ or 16:1. The area decreased by 1/16.

3.

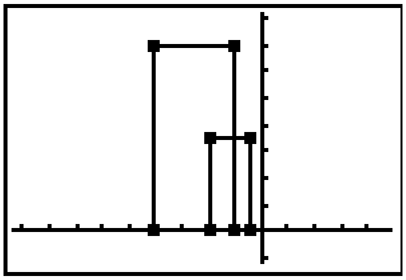


The vertices of the image are (-2, 8), (-2, -2), (6, -2).
The base of the image is the distance from (-2, -2) to (6, -2), 8 units.
The height of the image is the distance from (-2, 8) to (-2, -2), 10 units.
Area of a triangle = ½ bh, 40 square units.

The base of the pre-image is the distance from (-1, -1) to (3, -1), 4 units.
The height of the pre-image is the distance from (-1, 4) to (-1, -1), 5 units.
Area of a triangle = ½ bh, 10 square units.

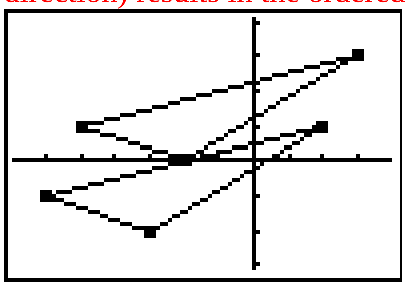
Probability of “hitting” the smaller triangle within the larger: 10/40 or 25%.

4. Performing the dilation results in ordered pairs (-0.5, 0), (-2, 0), (-2, 3.5), (-0.5, 3.5).

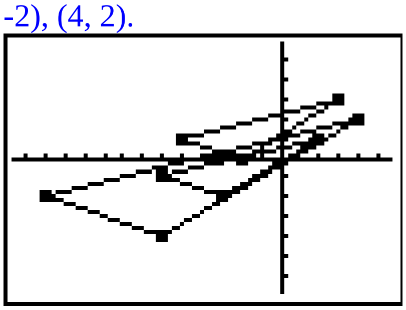


To reflect that (smaller) image over the y-axis, the y-coordinates remain the same and the x-coordinates are multiplied by -1, resulting in the coordinates (0.5, 0), (2, 0), (2, 3.5), (0.5, 3.5).

5. Performing the translation (one left = -1 in the x direction, two down = -2 in the y direction) results in the ordered pairs: (-3, -2), (-6, -1), (2, 1).



To dilate this image by a scale factor of two, multiply each coordinate by 2: (-6, -4), (-12, -2), (4, 2).



Quiz Yourself!

1. By what factor does the area of a triangle, whose vertices are (5, -5), (-4, -5), (5, 2), change when it is dilated by a scale factor of 5?
2. A rectangle with vertices (-20, 10), (-20, -10), (20, -10), (20, 10) is drawn on a large sheet of graph paper. The figure is dilated by a factor of ½ . What is the probability that a randomly thrown dart lands within the larger figure, but outside the smaller figure?
3. A trapezoid with vertices (1, -1), (-1, -1), (-1, 3), (1, 2) is translated 2 right and 4 up. It is then reflected over the x-axis and dilated by a scale factor of 2. What are the resulting ordered pairs?
4. Generalize: Write an inequality describing all scale factors which will produce a smaller image.

Check your answers:

1. The area increases by a factor of 25.
2. Area of larger rectangle = 20 x 40 = 800. Area of smaller rectangle = 20 x 10 = 200. 800-200 = 600. 600/800 = ¾ or 75%.
3. (6, -6), (2, -6), (2, -14), (6, -12)
4. $0<x<1$