Lesson: Pythagorean Theorem

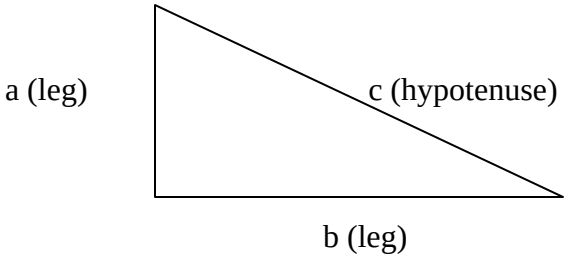
Eighth Grade Objective: 3.02: Apply geometric properties and relationships, including the Pythagorean theorem, to solve problems.

Lesson:
If you are given a square with side length 5 centimeters, how long are the other three sides? They are each 5 centimeters and we know this because in a square all the sides are the same length.

How about a rectangle? If you know the length is 3 inches and the width is 7 inches, how long are the other two sides? They are 3 inches and 7 inches and we know this because in a rectangle, the opposite sides have the same length.

In right triangles, there are also side length relationships we can discover and apply.

It is important to notice that this applies to RIGHT triangles. That means that one of the angles in the triangle has to be equal to 90 degrees. In a right triangle, the two sides that make up the right angle are called "legs" and the side that is across from the right angle is called the "hypotenuse".

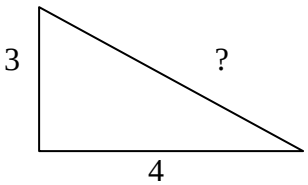


The Pythagorean theorem states that: in a right triangle, the sum of the squares of each of the legs is equal to the square of the hypotenuse. That means, if you square (raise to the second power) each of the legs and add them, you will get the square of the hypotenuse. Or, $a^2 + b^2 = c^2$.

Let's try:

Find the missing side length:

1.



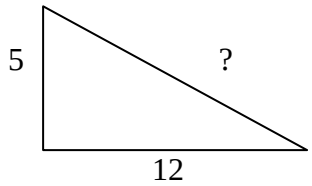
The sides labeled 3 and 4 are legs since they make up the right angle and the unknown is the hypotenuse. So we'll call the side labeled 3 'a', the side labeled 4 'b" and the unknown side is 'c'.

Using the Pythagorean theorem to solve for a missing length in a right triangle:

	$a^2 + b^2 = c^2$
Substitute:	$3^2 + 4^2 = c^2$
Follow order of operations:	$9 + 16 = c^2$
	$25 = c^2$
Undo the square by taking the square root:	$5 = c$

The length of the hypotenuse is 5 units.

2.



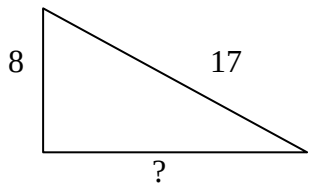
The sides labeled 5 and 12 are legs since they make up the right angle and the unknown is the hypotenuse. So we'll call the side labeled 12 'a', the side labeled 5 'b" and the unknown side is 'c'. (Notice that it does not matter which leg is a and which leg is b.)

Using the Pythagorean theorem to solve for a missing length in a right triangle:

	$a^2 + b^2 = c^2$
Substitute:	$12^2 + 5^2 = c^2$
Follow order of operations:	$144 + 25 = c^2$
	$169 = c^2$
Undo the square by taking the square root:	$13 = c$

The length of the hypotenuse is 13 units.

3.



The side labeled 8 is a leg since it makes up one side of the right angle, the other leg is unknown and hypotenuse is 17. So we'll call the side labeled 8 'a', the side labeled 17 'c' and the unknown side is 'b'.

Using the Pythagorean theorem to solve for a missing length in a right triangle:

	$a^2 + b^2 = c^2$
Substitute:	$8^2 + b^2 = 17^2$
Follow order of operations:	$64 + b^2 = 289$
Begin to solve the equation for b:	
First, subtract 64 from both sides	$b^2 = 225$
Take the square root to "undo" the square	$b = 15$

The length of the unknown leg is 15 units.

4. A right triangle has one leg 9 inches and the hypotenuse is 15 inches. How long is the other leg?

Call the 9 inch leg "b" and the 15 inch hypotenuse "c" (remember the hypotenuse is **always** c!)

Using the Pythagorean theorem to solve for a missing length in a right triangle:

	$a^2 + b^2 = c^2$
Substitute:	$a^2 + 9^2 = 15^2$
Follow order of operations:	$a^2 + 81 = 225$
Begin to solve the equation for b:	
First, subtract 64 from both sides	$a^2 = 144$
Take the square root to "undo" the square	$a = 12$

The unknown leg length is 12 units.

5. A firefighter needs to rescue a cat from a tree. The cat is 60 feet up the tree and the fireman knows that his ladder must be placed 11 feet from the base of the tree to be stable. How long must the fireman's ladder be to reach the cat safely?

Sketch a picture: start with the tree that has a cat near the top. Now place the ladder at a diagonal, starting at the ground (not at the base of the tree, but close, it's supposed to be 5 feet away). Notice that the ground and the tree make a right angle. Put labels on your diagram, the ladder is unknown and can be represented with a "c" since it is the hypotenuse. Label the distance from the tree to the base of the ladder "b = 5" and the height of the cat in the tree "a = 60". Now solve:

	$a^2 + b^2 = c^2$
Substitute:	$60^2 + 11^2 = c^2$
Follow order of operations:	$3600 + 121 = c^2$
	$3721 = c^2$

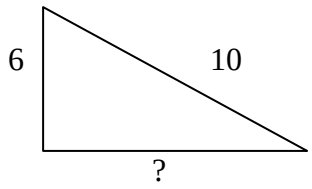
Undo the square by using

the square root: $61 = c$

The fireman's ladder must be at least 61 feet long.

Try these on your own:

1.



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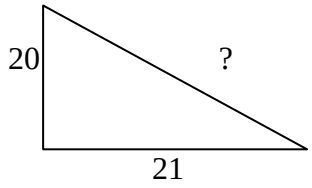


3. A tree is being anchored to the ground to ensure that it grows at a right angle with the ground. The wire that is being used to anchor the tree is 13 feet long and the anchor is attached to the tree 12 feet in the air. How far must the anchor be from the base of the tree?

Check your answers:

1. 8 units

$6^2 + b^2 = 10^2$
$36 + b^2 = 100$
$b^2 = 64$
$b = 8$

2. 29 units

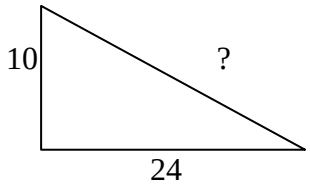
$20^2 + 21^2 = c^2$
$400 + 441 = c^2$
$c^2 = 841$
$c = 29$

3. 5 feet

$12^2 + b^2 = 13^2$
$144 + b^2 = 169$
$b^2 = 25$
$b = 5$

Quiz yourself:

1.



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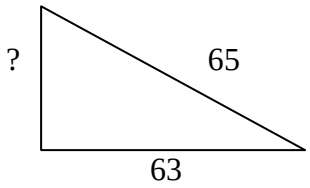

3. A ribbon is tied to the top of a pole and anchored to the ground 8 feet from the base of the pole. The 17 foot ribbon is stretched out fully. How tall is the pole?

Check your answers:

1. 26 units
2. 16 units
3. 15 feet

For online examples of the Pythagorean theorem, please see:

http://www.mathsisfun.com/pythagoras.html

For online practice, please see:

http://www.shodor.org/interactivate/activities/PythagoreanExplorer/?version=1.6.0-oem&browser=MSIE&vendor=Sun_Microsystems_Inc.