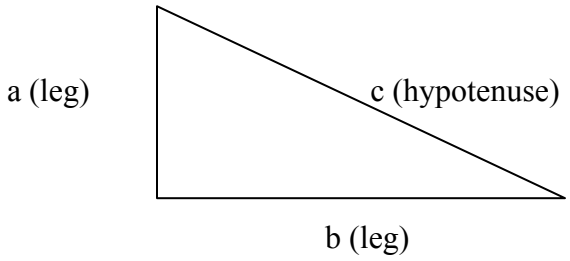
Enrichment Lesson: Pythagorean Theorem

Eighth Grade Objective: 3.02: Apply geometric properties and relationships, including the Pythagorean theorem, to solve problems.

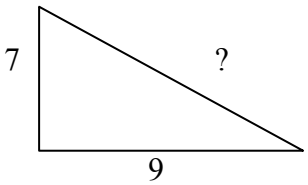
Lesson:
Review: The Pythagorean theorem can be used to determine missing side lengths in a right triangle.



The Pythagorean theorem states: In a right triangle, the sum of the squares of each of the legs is equal to the square of the hypotenuse. That means, if you square (raise to the second power) each of the legs and add them, you will get the square of the hypotenuse. Or, $a^2 + b^2 = c^2$.

Some basic examples:

Find the missing side length:
1.

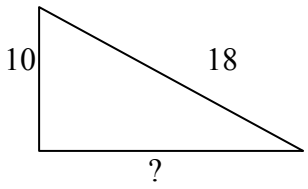


The sides labeled 7 and 9 are legs since they make up the right angle and the unknown is the hypotenuse. So we'll call the side labeled 7 'a', the side labeled 9 'b" and the unknown side is 'c'.
Using the Pythagorean theorem to solve for a missing length in a right triangle:

	$a^2 + b^2 = c^2$
Substitute:	$7^2 + 9^2 = c^2$
Follow order of operations:	$49 + 81 = c^2$
	$130 = c^2$
Undo the square by taking the square root:	the hypotenuse, c, is approximately 11.40 units
Or, write the answer in simplest radical form:	the hypotenuse is exactly $\sqrt{130}$ units in length

For assistance on simplest radical form, please visit:
http://www.themathpage.com/alg/simplify-radicals.htm#simplest

2.



The side labeled 10 is a leg since it makes up one side of the right angle, the other leg is unknown and hypotenuse is 18. So we'll call the side labeled 10 'b', the side labeled 18 'c' and the unknown side is 'a'.
Using the Pythagorean theorem to solve for a missing length in a right triangle:

	$a^2 + b^2 = c^2$
Substitute:	$a^2 + 10^2 = 18^2$
Follow order of operations:	$a^2 + 100 = 324$
Begin to solve the equation for a:	
First, subtract 100 from both sides	$a^2 = 224$
Take the square root to "undo" the square	the missing leg is approximately 14.97 units
Or, write the answer in simplest radical form:	the missing leg is exactly $4\sqrt{14}$ units in length

3. An isosceles right triangle has hypotenuse length 15 centimeters. What is the length of a leg of this triangle?

Using the Pythagorean theorem to solve for a missing length in a right triangle:

Note that in the problem, the triangle was described as isosceles, meaning both legs have the same length, therefore we can represent both legs with the same variable. Let's call both legs "a".

	$a^2 + a^2 = c^2$
Substitute:	$a^2 + a^2 = 15^2$
Follow order of operations:	$2a^2 = 225$
Begin to solve the equation for a:	
First, divide both sides by 2	$a^2 = 112.5$
Take the square root to "undo" the square	one leg length is approximately 10.61 centimeters

The converse of the Pythagorean theorem: if the dimensions of a triangle satisfy the equation $a^2 + b^2 = c^2$, then the triangle is a right triangle.

4. A triangle has dimensions 6, 8, 9. Is the triangle a right triangle?

Substitute the values into the Pythagorean Theorem: $a^2 + b^2 = c^2$

$6^2 + 8^2 = 9^2$

Simplify: $36 + 64 = 81$

$100 \neq 81$, therefore

the triangle is not a right triangle

Applications of the Pythagorean theorem:

5. The area of the sail on a boat is $60ft^2$. The base of the triangular sail is 10 feet long. How long is the hypotenuse?

Find the missing leg: $a = \frac{1}{2} bh$

$60 = \frac{1}{2} 10h$

$60 = 5h$

$12 = h$, or the other leg

Use the Pythagorean theorem:

$a^2 + b^2 = c^2$

Substitute: $10^2 + 12^2 = c^2$

Follow order of operations: $100 + 144 = c^2$

$244 = c^2$

Undo the square by using the square root:

the sail's hypotenuse is approximately 15.62 ft

the sail's hypotenuse is exactly $2\sqrt{61}$ ft

Try these on your own:

1. Maria is designing a play area in the shape of an isosceles triangle for her younger sister. The area of the triangle is 45 square feet and has a wooden border. The longest side of the triangle measures approximately 13.4 feet. How much wood does Maria need to purchase?

2. The area of a right isosceles triangle is 50 meters. How long is the hypotenuse of the triangle?

3. Caleb wants to ship a 7-foot curtain rod to his grandma. The largest box he has is a rectangular prism measuring 3 ft x4 ft x5 ft. Can he use the box to ship the curtain rod?

Check your answers:

1. In an isosceles triangle, both legs are the same length therefore we can represent both legs with the same variable. Let's call both legs "b".

$b^2 + b^2 = c^2$

Substitute: $b^2 + b^2 = 13.4^2$

Follow order of operations: $2b^2 = 179.56$

Begin to solve the equation for a:

First, divide both sides by 2 $b^2 = 89.78$

Take the square root to "undo" the square — one leg length is approximately 9.48 feet

To find the amount of wood needed: $9.48 + 9.48 + 13.4 = 32.36$ feet of wood

The information about area could have also been used to determine the missing side lengths.

2.

Find the length of each leg:

$a = \frac{1}{2} bh$

$50 = \frac{1}{2} bh$

This is an isosceles triangle, so b and h are the same, we can represent both with the same variable. Let's choose b.

$50 = \frac{1}{2} b*b$

$50 = \frac{1}{2} b^2$

$100 = b^2$

10 = b, since the triangle is isosceles, h = 10 also.

Use the Pythagorean theorem:

$a^2 + b^2 = c^2$

$10^2 + 10^2 = c^2$

$100 + 100 = c^2$

$200 = c^2$

c is approximately 14.14 or exactly $10\sqrt{2}$.

3. The rod obviously will not fit along an edge of the box, since the largest dimension is smaller than the length of the rod. Will the rod fit diagonally along the bottom?

The largest diagonal we could make across the bottom uses the two largest edges, 4 and 5. The hypotenuse of the right triangle with side lengths 4 and 5 is approximately 6.4 units. The curtain rod will not fit diagonally along the bottom of the box.

Will the rod fit "diagonally" from the back, upper corner to the opposite front, lower corner? To find this out, use the height of the box and the diagonal of the base as your legs and find the measure of the hypotenuse (an empty tissue box and a pencil might help you visualize this if you are having difficulties). The two legs then measure 3 and 6.4. Using the Pythagorean theorem, we find the length of this diagonal to be approximately 7.1. The 7-foot curtain rod will fit in a 3 ft x 4 ft x 5 ft box.

Quiz Yourself!

1. The hypotenuse of an isosceles right triangle measures 22 centimeters. What is the length of one of the legs?

2. Charla believes a triangle with side lengths 7, 24 and 25 is a right triangle. Is she correct and why?

3. What is the area of a right triangle whose hypotenuse measures 14 inches and the longer leg is twice the length of the shorter leg?

4. What is the smallest volume of a cube that a 10-foot curtain rod will be able to fit inside?

Check Your Answers:

1. 15.56 centimeters
2. Yes, she is correct, because the dimensions satisfy the Pythagorean theorem.
3. area = 39.19 square inches (use the equation $a^2 + (2a)^2 = c^2$, you'll find the side lengths to be 6.26 and 12.52.)
4. 192.45 cubic feet (the side length of the cube would be approx. 5.77. Use $s^2 + s^2 = d^2$ or $2s^2 = d^2$ for the measure of the diagonal of the base. Use that and the height of the cube (s^2) to find the measure of the diagonal of the box.)

For extensive proofs of the Pythagorean theorem, visit:
http://www.cut-the-knot.org/pythagoras/index.shtml