Lesson: Indirect Measurement

Eighth Grade Objective: **2.02 Apply and use concepts of indirect measurement.**

Lesson:
First, let's review two definitions. Congruent figures are figures whose angles are congruent and whose side lengths are equal (angles are the same, side lengths are the same). Similar figures are figures whose angles are the same, but their side lengths are proportional. We can use proportions to find measurements of within similar figures that are otherwise very hard to measure.

We'll start by determining if figures are congruent, similar or neither.

1.

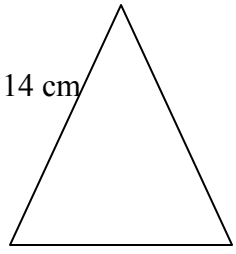
Even though all the angles in both figures are 90 degrees, the figures are not congruent, because the side lengths are not the same.
Let's set up proportions to determine if the figures are similar:

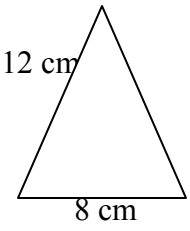
$$\frac{\text{Short side}}{\text{Long side}} \quad \frac{3 \text{ cm}}{5 \text{ cm}} = \frac{6 \text{ cm}}{10 \text{ cm}}$$

When we cross multiply, we get $3 * 10 \stackrel{?}{=} 5 * 6$

$30 = 30$, which is true, therefore the figures are similar, the angles are congruent, the sides are proportional.

2.





The figures are not congruent, because the side lengths are not the same. It is difficult to tell if the angle measures are congruent, so let's determine if the side lengths are proportional first:

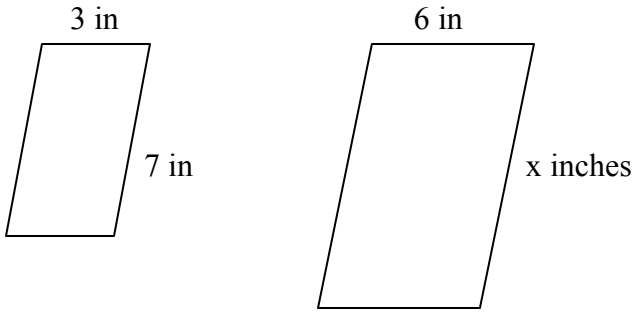
$$\frac{\text{Short side}}{\text{Long side}} \quad \frac{10 \text{ cm}}{14 \text{ cm}} = \frac{8 \text{ cm}}{12 \text{ cm}}$$

When we cross multiply, we get $12 * 10 \stackrel{?}{=} 8 * 14$

$120 \neq 112$, the sides are not proportional, so the figures are not similar. The figures are neither similar nor congruent.

We can also use indirect measurement to determine a missing side length in a pair of similar figures. In the below examples, in each pair of figures, the angles are congruent.

1.



Set up a proportion:

Short side	3 in	=	6 in
Long side	7 in		x in

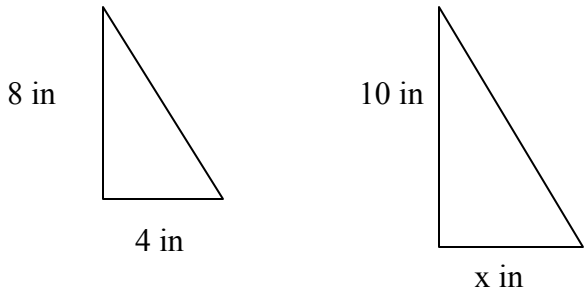
Cross multiply: $3x = 7 * 6$

$3x = 42$

$x = 14$

It is also possible to look at the multiplicative relationship between the corresponding sides of the figures, three times 2 is six, seven times 2 is fourteen, or x.

2.



Set up a proportion:

Short side	4 in	=	x in
Long side	8 in		10 in

Cross multiply: $8x = 4 * 10$

$8x = 40$

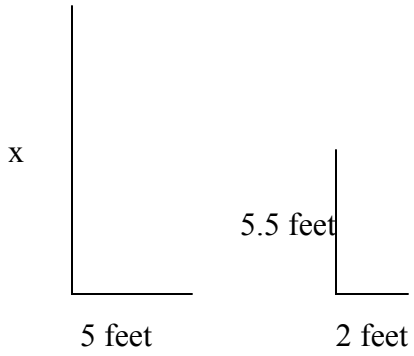
$x = 5$

It is also possible to look at the multiplicative relationship between the corresponding parts of the same triangles, in the first triangle, 4 is ½ of 8 so in the second x is ½ of 10, or 5 inches.

Similar figures are frequently used to determine measurements that would be hard to determine using other strategies. Most of us don't have access to the type of equipment that would let us measure the height of a flagpole or other tall object, or the distance across a large body of water. We can use other objects to determine properties of objects that are too complex for us to directly measure. This is called using indirect measurement.

1. A flagpole casts a five foot shadow. At the same time, a 5.5 foot tall girl casts a 2 foot shadow. How tall is the flagpole?

Start by drawing a picture:



Set up a proportion:

Short side	5 ft	=	2 ft
Long side	x ft		5.5 ft

Cross multiply: $2x = 5 * 5.5$

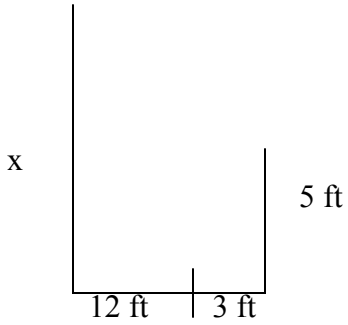
$2x = 27.5$

$x = 13.75$ feet

The flagpole is 13.75 feet tall.

2. A boy is determining the height of a building using mirrors. He places a mirror on the ground in front of the building and steps back until he can see the top of the building in the mirror. A friend measures the distance from the mirror to the base of the building (12 feet), the distance from the mirror to the boy's feet (3 feet) and the distance from the ground to the boy's eyes (5 feet). How tall is the building?

Start by drawing a picture:



Imagine two triangles drawn from the top of each vertical line to the small vertical line representing the mirror. We now have two similar triangles.

Set up a proportion:

Short side	12 ft	=	3 ft
Long side	x ft		5 ft

Cross multiply: $3x = 5 * 12$

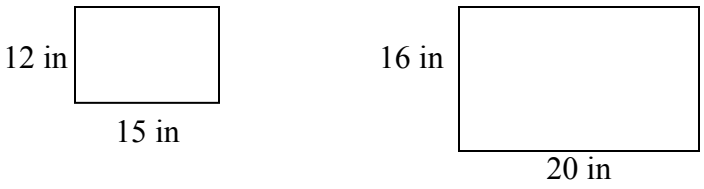
$3x = 60$

$x = 20$ feet

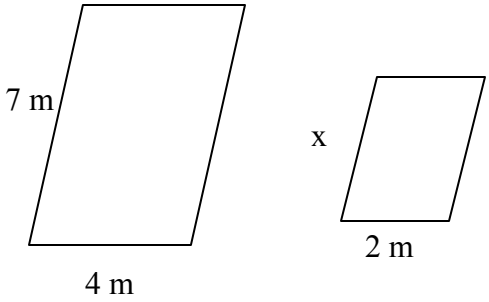
The building is 20 feet tall.

Try these on your own!

1. Use proportions to determine if the figures are similar, congruent or neither.



2. Determine the missing side length. All angles are congruent.



3. Megan is trying to determine the height of the ceiling in her house. She places a mirror on the ground and steps back until she can see where the wall meets the ceiling. A friend measures and determines she is standing 2 feet from the mirror, her eyes are 5.25 feet from the ground and the mirror is 5 feet from the base of the wall. How tall is the ceiling?

Check your answers:

1. Set up a proportion:

$$\frac{\text{Short side}}{\text{Long side}} \qquad \frac{12 \text{ ft}}{15 \text{ ft}} = \frac{16 \text{ ft}}{20 \text{ ft}}$$

Cross multiply:

$12 * 20 \stackrel{?}{=} 15 * 16$

$240 = 240$, yes, the figures are similar.

2. Set up a proportion:

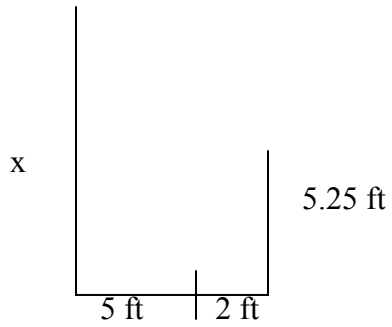
$$\frac{\text{Short side}}{\text{Long side}} \qquad \frac{4 \text{ m}}{7 \text{ m}} = \frac{2 \text{ m}}{x \text{ m}}$$

Cross multiply:

$4x = 2 * 7$

4x = 14
x = 3.5 meters
You could have also used the relationship 2 is ½ of 4, so x must be ½ of 7.

3.
Start by drawing a picture:



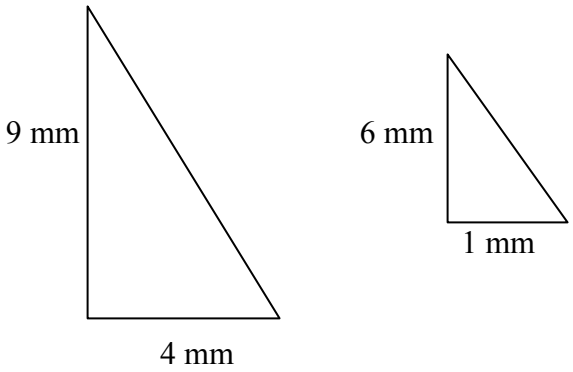
Set up a proportion:

Short side	5 ft	=	2 ft
Long side	x ft		5.25 ft

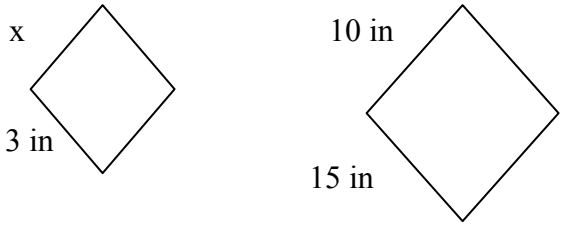
Cross multiply:
5 * 5.25 = 2x
26.25 = 2x
13.125 = x, the ceiling is 13.125 feet tall

Quiz yourself!

1. The following triangles have congruent angles, are the triangles similar?



2. The following figures are proportional, what is the length of the missing side?



*not drawn to scale

3. Will is measuring the height of his school building. At noon, the school casts a shadow that is 4 feet long. At the same time, Will casts a shadow that is 1.5 feet long. If Will is 6 feet tall, how tall is the school building?

Check your answers:

1. No, the triangles are not similar. The sides must be proportional for the figures to be similar (that means related by multiplication, not addition!).
2. The missing side length is 2 inches.
3. The school building is 16 feet tall.