

Lesson: Indirect Measurement

Eighth Grade Objective: 2.02 Apply and use concepts of indirect measurement.

Lesson:

Through the use of similar figures and indirect measurement, we are able to find measurements that would otherwise prove very difficult or impossible.

Before proportions can be used to determine missing lengths, we must first determine if figures are similar. Similar figures have congruent angles and proportional sides.

Can the following triangles be similar?

1. Triangle A: angle measures 45° and 52° , Triangle B: angle measures 83° and 45° .

Yes, the triangles could be similar because the angle measures of the first triangle are 45° and 52° and $180^\circ - (45^\circ + 52^\circ)$, which is 83° . The second triangle has angles 45° and 83° and $180^\circ - (45^\circ + 83^\circ)$, which is 52° . The triangles have congruent angles and can therefore be similar. The triangles could also be congruent, we do not have enough information to determine that.

2. Triangle C: isosceles triangle right triangle, Triangle D: a right triangle with side lengths 10 inches, 10 inches and $10\sqrt{2}$ inches.

Yes, the triangles could be similar, for the triangle to be isosceles, not only are two sides congruent, but their opposite angles are congruent. Because triangle C must have angle measures of 90° and each of the other angles must have measure: $(180^\circ - 90^\circ) / 2$, we have a 45° - 45° - 90° triangle. Triangle D is also a 45° - 45° - 90° because it is stated to be a right triangle with two congruent sides. Therefore the triangles can be similar. The triangles could also be congruent, we do not have enough information to determine that.

3. Triangle E: scalene triangle with side lengths 6 mm, 8 mm, and 10 mm, Triangle F: a scalene triangle with side lengths 8 in, 10 in, and 12 in.

No, the triangles are not similar because their side lengths are not proportional. Take the smallest side over the largest side in each triangle: $6/10$ and $8/12$. When simplified, it is clear that the relationship between the longest and shortest sides in each triangle is different, therefore the triangles are not similar.

Note that when determining if triangles are similar, it is sufficient to determine that either the sides are proportional or that the angles are congruent. This is not true with other shapes, such as rectangles.

Can the following figures be similar?

1. Rectangle A: all four sides congruent, Rectangle B: square.

Yes, the rectangles could be similar. All rectangles have four 90° angles and, in this case, each rectangle has a 1:1 ratio between sides within each figure, making the angles

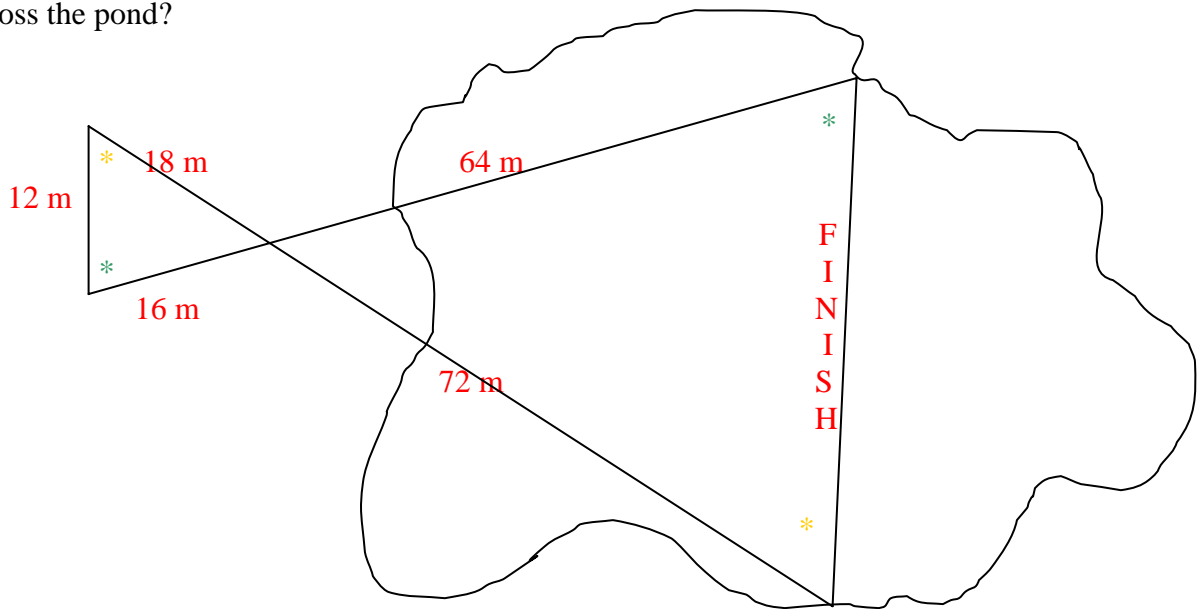
of the rectangles congruent and the sides proportional. The figures could also be congruent, we do not have enough information to determine that.

2. Rectangle C: $w = 2l + 3$, Rectangle D: $w = 13, l = 5$.

In this case, the rectangles may be congruent, but they are not similar. Other lengths and widths that satisfy the equation include $w = 15, l = 6$; $w = 17, l = 7$; you'll notice the arithmetic pattern (you may also see the increase as a consistent slope). Though there is a pattern, proportionality is a multiplicative relationship, not an additive relationship. Though the angles are certainly congruent based on the definition of a rectangle, the rectangles are not similar.

We can use indirect measurement to determine things not easily measured directly. The height of most buildings, distance across large bodies of water, diameters of very large objects are not easy to take out a ruler and measure. In order to measure these, we can use shadows, mirrors and other measurements that are easier to take to determine the more challenging measures.

1. Karissa is responsible for stretching out a piece of ribbon across a pond as a finish line for a boat race. Once she is on the water, she doesn't want to have to get back out to get more ribbon, how much ribbon does she need to have in the boat to stretch the whole way across the pond?



Remember that vertical angles are congruent, so the two angles where the triangles meet are congruent. Alternate interior angles are also congruent, so the angles marked with a * are congruent, as are the angles marked with *. Knowing this, we can determine that the corresponding sides are 18 and 72, 16 and 64, 12 and x.

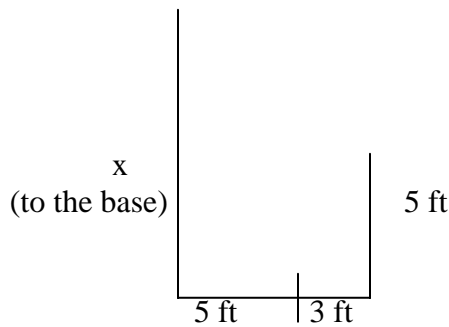
We can set up proportions using any of this information:

$$\frac{18 \text{ m}}{72 \text{ m}} = \frac{12 \text{ m}}{x \text{ m}}$$

After cross-multiplying, we arrive at $x = 48$ meters. Karissa needs 48 meters of ribbon for the finish line.

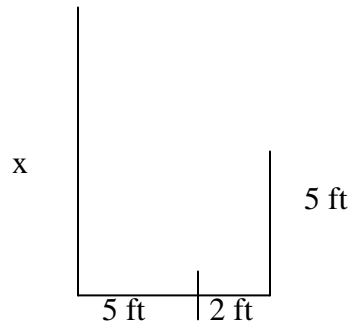
2. Jennifer is determining the area of a rectangular window that is high off the ground. She places a mirror on the ground between herself and the base of the building and positions herself so that she can see the base of the window. A friend measures the distance between Jennifer and the mirror (3 feet), the mirror and the base of the building (5 feet). Jennifer then repositions herself so that she can see the top of the window. She is now 2 feet from the mirror. If Jennifer's eyes are 5 feet off the ground, what is the area of the window if the width of the window is 3 feet?

First, draw a picture:



$$\frac{5 \text{ ft}}{3 \text{ ft}} = \frac{x \text{ ft}}{5 \text{ ft}}$$

The base of the window is $8 \frac{1}{3}$ feet off the ground.



$$\frac{5 \text{ ft}}{2 \text{ ft}} = \frac{x \text{ ft}}{5 \text{ ft}}$$

The top of the window is $12 \frac{1}{2}$ feet off the ground.

The height of the rectangular window is $12 \frac{1}{2} - 8 \frac{1}{3} = 4 \frac{1}{6}$ feet. The width of the window is 3 feet. $a = lw$, $a = 3 * 4 \frac{1}{6}$, $a = 12.5$ square feet.

Try these on your own!

- Can the following triangles be similar?
Triangle R: equilateral triangle
Triangle Q: has two 60° angles
- Can the following rectangles be similar?
Rectangle M: $l = w^2$
Rectangle K: $w = 3, l = 9$

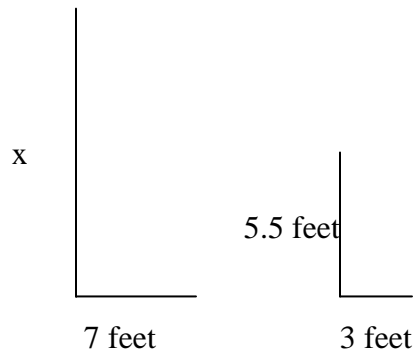
3. A large tree is being anchored to the ground to ensure that it grows perpendicular to the ground. The anchor is fixed to the tree 2 feet from the top of the tree to a point on the ground 10 feet from the base of the tree. At 10 am, 5 foot 6 inch Loren observes the tree's 7 foot shadow and measures her own shadow at 3 feet long. A) How tall is the tree? B) How long is the anchor wire?

Check your answers:

1. Yes, the triangles can be similar. Remember, with triangles you only have to determine if the angles are congruent OR the sides are proportional. In all equilateral triangles the angles are congruent to each other, they all have degree measure $180/3 = 60$. Triangle Q has two 60 degree angles, which means the third must be $180 - 60 - 60 = 60$ degrees. Both triangles have three 60 degree angles. The triangles may also be congruent.
2. The rectangles may be congruent, but they are not similar. Other examples of lengths and widths that satisfy the requirements for Rectangle M are: $l = 16, w = 4$; $l = 25, w = 5$. Expressing these as ratios, we have $16/4 = 4$, $25/5 = 5$ and $9/3 = 3$.
3. While there is a pattern, the rectangles are not similar.

3.

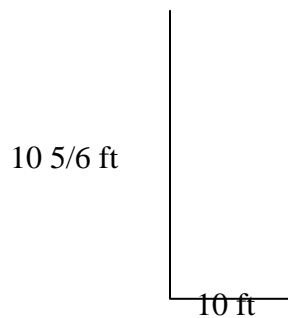
Start by drawing a picture:



$$\frac{5.5 \text{ ft}}{x \text{ ft}} = \frac{3 \text{ ft}}{7 \text{ ft}}$$

The tree is $12 \frac{5}{6}$ feet tall.

The anchor wire is attached 2 feet from the top of the tree, so it is attached at $10 \frac{5}{6}$ feet or 10 feet 10 inches off the ground and 10 feet from the base of the tree.



Using the Pythagorean theorem, we can determine the distance:

$$a^2 + b^2 = c^2$$

$$100 + 117.4 = c^2$$

$$217.4 = c^2$$

$$14.7 = c$$

The anchor wire is approximately 14.7 ft long.

Quiz Yourself!

1. Could the following triangles be similar? Give an example or counterexample as necessary.

Triangle H: Scalene right triangle

Triangle Y: Triangle with sides 5, 12, 13

2. Could the following rectangles be similar?

Rectangle V: $l = 2w$

Rectangle X: $l = 4, w = 2$

3. A triangular piece of cloth is to be extended from the top of the flagpole to the ground five feet from the base of the flagpole. At 3 pm, Kristin's friend measures the shadow of the flagpole (20 feet) and Kristin's shadow (8 feet). If Kristin is 5 feet 3 inches tall, what is the area of cloth that must be purchased?

Check your answers:

1. The triangle could be similar, a triangle with side lengths 5, 12 and 13 is scalene and it is right (use the Pythagorean Theorem to test this). A triangle with side length 10, 24, 26 is also scalene and right and similar to the first example. However, another example of a scalene right triangle is a triangle with side lengths 3, 4, 5. It is scalene, it is right, but it is not similar to triangle Y.
2. The rectangles can either be congruent or similar. Other examples of dimensions include: $l = 6, w = 3$; $l = 8, w = 4$; all of which have a common ratio of 2.
3. The height of the flagpole is 13.125 (13 $\frac{1}{8}$) feet tall. The area of the cloth must be 32.8125 square feet.