Enrichment Lesson: How do changes in dimensions of objects affect their perimeter, area and volume?

Eighth Grade Objective: **2.01** Determine the effect on perimeter, area or volume when one or more dimensions of two- and three-dimensional figures are changed.

Lesson:

By changing the dimensions of two- and three-dimensional objects, we can change the perimeter, area and volume in ways that can be predicted using algebra.

Take a rectangle, for example. To find the perimeter of a rectangle, we would add each of the sides: p = l + l + w + w …or… p = 2l + 2w. If we double each dimension, the length becomes 2l and the width becomes 2w. Substitute these values into the original equation and we arrive at: p = 2(2l) + 2(2w) …or… p = 2(2l + 2w). We have increased the original perimeter by a factor of 2.

Perimeter of Rectangles
Original Formula: p = 2l + 2w
New Formula (both dimensions doubled): p = 2(2l+2w)

The formula for finding the area of a rectangle is a=lw. If we are going to double both dimensions, we can represent the new length as 2l and the new width as 2w. The area of the new figure would be a = (2l)(2w) or a = 4lw. We have increased the original area by a scale factor of four.

Area of Rectangles:
Original Formula: a = lw
New Formula(both dimensions doubled): a = 4lw

We can apply these principals to any type of scalar change.

Let's try using a triangle.

To find the perimeter of a triangle, add the lengths of the sides: $p = s_1 + s_2 + s_3$. If we triple each of the sides, we can represent each of the new sides by 3s. Substitute this into the original equation and we arrive at: $p = 3s_1 + 3s_2 + 3s_3$ …or… $p = 3(s_1 + s_2 + s_3)$. We have increased the original perimeter by a factor of 3.

The formula for finding the area of a triangle is a = ½ bh. If we are going to triple both dimensions, we can represent the new base as 3b and the new height as 3h. The area of the new figure would be a = ½ (3b)(3h) …or…a = ½ * 9 (bh). We have increased the original area by a scale factor of nine.

Volume works similarly. The formula for finding the volume of a rectangular prism is v = lwh. If we increase each side by a scale factor of 2, we can rewrite our equation as v = (2l)(2w)(2h) …or… v = 8 lwh. The volume has increased by a scale factor of 8.

Try these on your own:

1. By what scale factor does the perimeter and area of a circle increase if the radius is tripled?
The perimeter of a circle is its circumference or $c = 2\prod r$. If the radius is doubled, we have $c = 2\prod(2r)$. The perimeter has increased by a factor of 2.
The area of a circle is $a = \prod r^2$. If the radius is doubled, we have $a = \prod(2r)^2$, or $\prod 4r^2$ (remember to raise everything in the parentheses to the stated power!). We have increased the area of the circle by a factor of four.

2. By what scale factor does the area of a rectangle increase when the length is tripled?

The area of a rectangle is found by $a = lw$. If the length is tripled, we substitute $3l$ for l and arrive at the equation $a = 3lw$. The area of the rectangle has been increased by a factor of three.

3. A triangle's area has increased by a factor of 4. What are two distinct ways this is possible?

The area of a triangle is $a = ½\ bh$. The area of the new triangle is $a = ½ * 4\ (bh)$. This can be accomplished by increasing the base by a factor of four, increasing the height by a factor of four or increasing both the base and the height by two. (Note: there are infinite other possibilities: ex. Multiplying the base by 8 and multiplying the height by ½).

4. The perimeter of a circle has increased by a factor of 4. By what factor did the area increase?

Originally, $p = 2\prod r$. This became $p = 2\prod * 4 * r$. We have learned that when dimensions are changed by a uniform scale factor, the perimeter increases by the same factor. The converse is true also, therefore the scale factor applied to the original circle was 4.

If the original radius increased by a scale factor of four, we apply this to the area formula: $a = \prod r^2$. This becomes $a = \prod(4r)^2$ or $a = \prod * 16 * r^2$. The area increases by a scale factor of 16.

5. By what factor does the volume of a cylinder increase if the radius is doubled and the height is tripled?

Originally, $v = \prod r^2 h$. If the radius is doubled and the height is tripled, we have $v = \prod(2r)^2 * 3h$. Simplified, this is $v = \prod 4r^2 * 3h$ or $\prod * 12 * r^2 h$. The volume increases by a factor of 12.

6. Which has a greater impact on the volume of a cylinder: tripling the radius or tripling the height? Why?

Originally, $v = \prod r^2 h$.

If the radius is tripled, we have $v = \prod(3r)^2h$ or $v = \prod*9*r^2h$. The volume increases by a factor of 9.

If the height is tripled, we have $v = \prod r^2 * 3h$. The volume increases by a factor of 3.

Tripling the radius has a greater impact on the volume of a cylinder.

Quiz Yourself:

1. By what factor does the perimeter and area of a rectangle increase by if both dimensions are increased by a factor of five?

2. By what factor does the area of a triangle increase if the height is doubled?

3. The area of a circle is increased by a scale factor of 9. Is there more than one way in which this can occur? Either give multiple examples or state why it is not possible.

4. A candy box is being redesigned. The original dimensions are l, w and h. Which new rectangular candy box would be hold the most candy:
   Box A: Each dimension is doubled.
   Box B: The length and the width are tripled and the height is halved.
   Box C: The length is doubled, the width is halved, and the height is quadrupled.

5. The base area of a cylinder is increased by a factor of 4. The height is doubled. By what factor does the volume increase?

Check your answers:

1. The perimeter increases by a factor of 5 and the area increases by a factor of 25.
2. The area increases by a factor of two.
3. The only way the area of a circle can increase by a factor of nine is to have the radius increase by a factor of 3. There is only one dimension and that dimension is squared. The only positive number that can be squared and result in nine is three.
4. Box A holds the most candy (8 times as much as the original)
5. The volume increases by a factor of 8. $Bh = v$ so $4B*2h = v$ or $8Bh = v$.