Lesson: The Real Number System

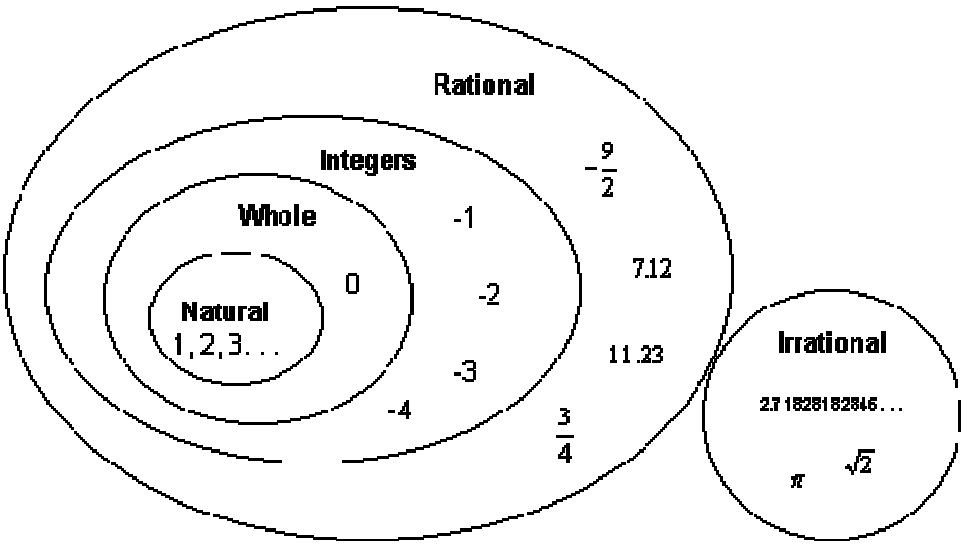
Eighth Grade Objective: 1.01 Develop number sense for the real numbers.

a. Define and use irrational numbers.
b. Compare and order.
c. Use estimates of irrational numbers in appropriate situations.

Lesson:

The Real Number System includes any and all numbers that fit into the following categories:

1. Natural Numbers (Counting Numbers): These are the numbers that you would use if you were playing hide and go seek with a three year old: 1, 2, 3, 4, …
2. Whole Numbers: These are the Natural Numbers and zero: 0, 1, 2, 3, 4, …
3. Integers: These are Whole Numbers and their opposites: …-3, -2, -1, 0, 1, 2, 3…
4. Rational Numbers: These are any numbers that can be written as a fraction, a/b, where a and b are both integers and b is not equal to zero.
5. Irrational Numbers: These numbers cannot be written as a fraction, for example, non-repeating, non-terminating decimals.



Notice in the diagram, that if a number falls into the "Natural" circle, it is also within the "Whole" circle, the "Integer" circle and the "Rational" circle. The number 3, for instance, can be classified as natural, whole, integer, rational and real.

Using the diagram above, classify the following numbers:

1. -4

-4 is an integer, rational and real. It is the opposite of a whole number and can be written as -4/1.

2. 15/2
15/2 is rational and real.

3. 25
25 is natural, whole, integer, rational and real.

4. -1.23149…
-1.23149… is irrational and real. Notice the ellipses at the end of the number. That means that the number continues but it is not repeating. If the number is repeating, it will be denoted by a repeating bar above the appropriate part of the fraction.

5. 147.5
147.5 is rational and real.

6. ð9
ð9 is the same as 3, therefore it is natural, whole, integer, rational and real.

7. ð3
ð3 is 1.7320508…, it does not repeat and it does not end, therefore the square root of three is irrational and real.

It's important to be able to determine where numbers are in relation to other numbers. For example, ∏ is an irrational number that is between 3 and 4, closer to 3. When ordering numbers, it is usually easiest to put all of the numbers in the same form. If you are more comfortable with decimals, convert all the numbers that you need to order to decimals. If you are more comfortable with fractions, convert all the numbers that you need to order to fractions.

Order the following from least to greatest:

1. -4/5, -1.7, 0, -5/4, -ð4
As decimals, the numbers are:
-4/5 = -0.80
-1.7 = -1.70
0 = 0.00
-5/4 = -1.25
-ð4 = -2.00
Notice that I have lined up all my decimals vertically. Now I look for the smallest number. It is -2, which is -ð4. The next smallest is -1.7. The next smallest is -1.25, which is -5/4. The next smallest is -0.80, which is -4/5. And our largest number is 0. Remember when you are ordering decimals, begin by comparing the left most place value. Don't forget to line up your decimals!
Our final order is: -ð4, -1.7, -5/4, -4/5, 0. Notice that we write our final answer using the numbers in the problem.

2. 3.5, ∏, ð12, 11/3, 4

As decimals, the numbers are:
3.5 = 3.50
∏ ≈ 3.14
ð12 ≈ 3.46
11/3 ≈ 3.67
4 = 4.00
Starting with the smallest number, we have 3.14, which is ∏. Then 3.46, which is ð12. Then 3.5. Next is 3.67, which is 11/3. And our largest number is 4.
Our final order is ∏, ð12, 3.5, 11/3, 4.

Sometimes, you'll be asked to approximate the value for an irrational number without using a calculator. Here's a strategy for doing such if your irrational number is a square root:

What two whole numbers is ð44 between?

Make a list, mental or you can write it down, of perfect squares.

x	1	2	3	4	5	6	7	8	9	10	11
x^2	1	4	9	16	25	36	49	64	81	100	121

Now, take the number that is under the radical sign, in our case 44, and figure out where it fits on the bottom row of our perfect squares chart. It fits between 36 and 49. Now look above the 36 and 49. The square root is between 6 and 7.

Use the table to determine which two whole numbers the following square roots are between:

1. ð52
52 fits between 49 and 64, so the square root of 52 is between 7 and 8.

2. ð2
2 fits between 1 and 4, so the square root of 2 is between 1 and 2.

3. ð91
91 fits between 81 and 100, so the square root of 91 is between 9 and 10.

Problem Solving using square roots:

1. The area of a square tablecloth is 56 square feet. Estimate the side length of largest square table the table cloth can cover?

To find the area of a square, $a = s^2$. So, $56 = s^2$. To undo the square, use the square root. ð56 is between 7 and 8, so the largest square table the table cloth could cover is between 7 and 8 feet.

2. The area of a square wall is 108 square feet. Estimate the height of the wall.

$a = s^2$, therefore $108 = s^2$. To undo the square, we take the square root and the square root of 108 falls between 10 and 11. The wall is between 10 and 11 feet tall.

Try these on your own!

Which sets of numbers do the following belong?

1. -14
2. ð25
3. 12.7253…
4. 12.72
5. 0

Order the following from least to greatest:

6. ð15, 16/5, 4.1, ð16
7. -7, -ð63, -8, -65/8
8. ð3, ∏, 3.14, 22/7

Problem Solving:

9. A square picture has area 47 inches squared. What is the approximate length of one side of the picture?
10. A square monitor has area 135 inches squared. What is the approximate length of one side of the monitor?

Check your answers:

1. integer, rational, real
2. natural, whole, integer, rational, real (that simplifies to 5)
3. irrational, real
4. rational, real (it can be written as a fraction 318/25)
5. whole, integer, rational, real
6. The numbers as decimals:
   ð15 ≈ 3.87
   16/5 = 3.20
   4.1 = 4.10
   ð16 = 4.00
   The smallest number is 3.20 or 16/5. The next is 3.87 or ð15. The next is 4.00, or ð16. And the largest number is 4.1.
   In order: 16/5, ð15, ð16, 4.1.
7. The numbers as decimals:
   -7 = -7.00
   -ð63 ≈ -7.94
   -8 = -8.00

-65/8 = -8.125
The smallest number is -8.125, or -65/8. The next is -8. The next is -7.94, or -ð63. The largest number is -7.
In order: -65/8, -8, -ð63, -7.

8. As decimals:
   ð3 ≈ 1.732
   ∏ ≈ 3.142
= 3.140
   22/7 ≈ 3.143
   The smallest number is 1.732, or ð3. The next is 3.14. The next smallest is 3.142 or ∏. And the largest number is 3.143, or 22/7.
9. $47 = s^2$. The square root of 47 is between the square root of 36 and the square root of 49. That means the square root of 47 is between 6 and 7.
10. $135 = s^2$. The square root of 135 is between the square root of 121 and the square root of 144. That means the square root of 135 is between 11 and 12.

Quiz Yourself!

Which sets of numbers do the following belong?

1. - ð30
2. 12

Order the following from least to greatest:

3. – ð24, -9/2, -5, -4.82
4. ð27, 5.76, 26/5, 5

Problem Solving:

5. What is the approximate side length of a square table cloth whose area is 39 square feet?

Check your answers:

1. irrational, real
2. natural, whole, integer, rational, real
3. -5, – ð24, -4.82, -9/2
4. 5, ð27, 26/5, 5.76
5. The side length is between 6 and 7 feet.