Objective 4.03 Solving Systems using elimination:

Another way of solving systems of equations is by elimination. In elimination you "eliminate" one of the variables. Doing this allows you to solve for the other variable.

How to solve linear systems using the elimination method:

1. Make sure each equation is in standard form
2. Determine which variable can be eliminated, if neither then go to step three, if one can then skip to step 4.
3. Choose the easiest to eliminate. Sometimes you may just have to multiply one equation by a number. Sometimes you will have to multiply both. If one pair already has opposite signs, then that one will be a little easier. Always remember to multiply the WHOLE equation.
4. Combine the variable and your constants.
5. Solve.
6. Use the variable you solved for to substitute in either original equation to get the other one
7. Write your answer as an ordered pair.

Example 1:

$x + y = 7$ *If you combine these two equations then the y will be
$x - y = 9$ eliminated because they are opposites.

$x + y = 7$ *Add the x's and constants.
$\underline{x - y = 9}$
$\frac{2x}{2} = \frac{16}{2}$ *Solve for x

$x = 8$ *Now you can substitute 8 for x in either equation.

$x + y = 7$ *Substitute 8 for x in the first equation and solve for y
$8 + y = 7$
$\underline{-8 \quad -8}$
$y = -1$

The solution to the system is (8, -1)

Let's try another one:

$6x + 12y = -5$ *If you combine these two equations neither variable will cancel.
$6x + 9y = -3$ In systems like this, you must decide what you can do to make one of the variables be eliminated.

*Multiplying either equation by a -1 will eliminate x

$-1(6x + 12y = -5)$ *Multiply everything by a -1

$-6x - 12y = 5$ *This is your new equation. Combine it with original second equation.
$\underline{6x + 9y = -3}$ *Now your x's are eliminated

$$\frac{-3y}{-3} = \frac{2}{-3}$$

$y = -2/3$ *Substitute in -2/3 for y in either original equation

$6x + 12y = -5$
$6x + 12(-2/3) = -5$
$6x - 8 = -5$
$\underline{\quad +8 \quad +8}$

$$\frac{6x}{6} = \frac{3}{6}$$

$x = 1/2$
The solution to the system is (1/2, -2/3)

Example 3:

$3x + 4y = -25$
$2x - 3y = 6$

*To solve this equation by elimination you have to multiply BOTH equations by something in order to get one of the variables to eliminate.

Lets think for a minute…what could you multiply each equation by to get x to cancel? If we multiply the top by 2 and the bottom by 3, that would make each of them 6. Remember that they need to be opposite signs as well, so either the 2 or 3 would have to be negative. If you wanted to eliminate y, then you could multiply the top by 3 and the bottom by 4—one of them would have to be negative. Let's eliminate x:

$-2(3x + 4y = -25)$	becomes	$-6x - 8y = 50$	*Now you are ready to solve.
$3(2x - 3y = 6)$	becomes	$6x - 9y = 18$	

$-6x - 8y = 50$
$\underline{6x - 9y = 18}$

$$\frac{-17y}{-17} = \frac{68}{-17}$$

$y = -4$ Now substitute -4 in for y in either original equation

$3x + 4y = -25$
$3x + 4(-4) = -25$
$3x - 16 = -25$
$\underline{\quad +16 \quad +16}$

$$\frac{3x}{3} = \frac{-9}{3}$$

$x = -3$
The solution of the system is (-3,-4)

**Remember there will sometimes be no solution or infinitely many solutions*

$$\begin{array}{rcl} x - y &=& 2 \\ \underline{-x + y} &\underline{=}& \underline{-2} \\ 0 &=& 0 \end{array}$$

*If you have a system that ends up with a true statement, the answer is infinitely many solutions.

$$\begin{array}{rcl} 2x + 4y &=& 8 \\ \underline{-2x - 4y} &\underline{=}& \underline{8} \\ 0 &=& 16 \end{array}$$

*If you have a system that ends up with a false statement, the answer is no solution.

You try:

1. $x - y = 6$
   $x + y = 5$

2. $x + 2y = 8$
   $3x + 2y = 6$

3. $3x + 5y = 11$
   $2x + 3y = 7$

4. $3x + 2y = 14$
   $x + 2/3\ y = 4$

5. $y = 3x$ *Hint: Subtract 3x and set equation equal to 0
   $x + 2y = -21$

6. $x = 3 - 2y$
   $2x + 4y = 6$

Answers: 1. (11/2, -1/2) 2. (-1, 9/2) 3. (2,1) 4. no solution 5. (-3, -9) 6. infinitely many solutions