

Objective 4.01 Linear Equations

A linear equation is an equation of a LINE. A line can be determined by any two points that lie on it.

There are three common forms of linear equations.

- I. Slope-Intercept Form
- II. Point-Slope Form
- III. Standard Form

I. Slope-Intercept Form

$$y = mx + b$$

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slope y-intercept

The *y-intercept* is where the line crosses the y-axis.

Examples:

- a. Write an equation of the line with a slope of -1 and y-intercept of 2 .

The question gives you the following information: $m = -1$ $b = 2$
Put the values into the formula:

$$y = -1(x) + 2$$
$$y = -x + 2$$

- b. Find the slope and y-intercept: $3 + 2x + y = 0$

Write the equation given in slope intercept form:

Get y by itself

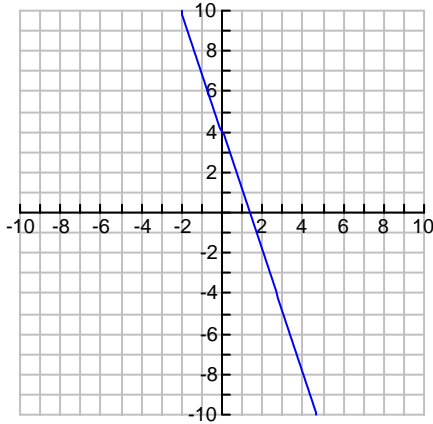
$$3 + 2x + y = 0$$

$$\begin{array}{r} -3 \quad -3 \\ \hline 2x + y = -3 \end{array}$$

$$\begin{array}{r} -2x \quad -2x \\ \hline y = -2x - 3 \end{array}$$

$$m = -2 \quad b = -3$$

c. Write the equation of the line from the given graph:



* To find the slope you can use the points (0, 4) and (1, 1). Starting at the point (1, 1) count up 3 and left 1 to get to (0, 4). This gives you a slope of -3.

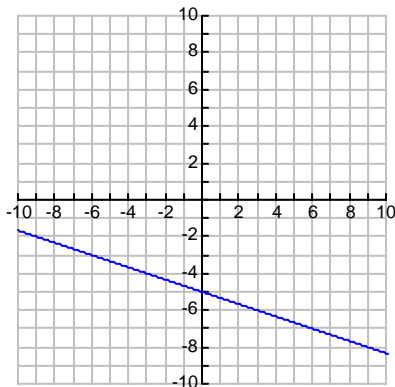
* The b is where the line crosses the y-axis.

$$m = -3 \quad b = 4$$

$$y = -3x + 4$$

You try:

1. Write an equation of a line with a slope of -3 and a y-intercept of -2.
2. Find the slope and y-intercept: $4x - 8y + 16 = -8$
3. Write the equation of the line from the given graph



Answers: 1) $y = -3x - 2$ 2) $y = \frac{1}{2}x + 3$ 3) $y = -\frac{1}{3}x - 5$

II. Point-Slope

When you are given the slope of a line and a point on the line, you can use a formula to find the equation of the line. The formula is called the point-slope formula.

Point-slope formula:

$$y - y_1 = m(x - x_1).$$

↑ ↑ ↑
y-coordinate slope x-coordinate

Examples:

a. Determine the point and slope of the following equation:

$$y - 2 = 3(x + 4)$$

This can be written as $y - 2 = 3(x - -4)$ because there is a subtraction sign in the formula.

The point is (2, -4), and the slope is 3.

b. Write the equation of the line with a slope of -2 that passes through the point (1,-3)

You are given a POINT and a SLOPE, so you can use POINT-SLOPE!

$$y + 3 = -2(x - 1) \quad \text{What if we wanted to change this to Slope-intercept Form?}$$

$$y + 3 = -2x + 2 \quad \text{(Distribute)}$$

$$\begin{array}{r} -3 \quad \quad -3 \\ \hline y = -2x - 1 \end{array}$$

c. Write the equation of the line in slope-intercept form.

$$y + 5 = -\frac{1}{3}(x - 2)$$

$$y + 5 = -\frac{1}{3}x + \frac{2}{3} \quad \text{(Distribute)}$$

$$\begin{array}{r} -5 \quad \quad -5 \\ \hline y = -\frac{1}{3}x - \frac{13}{3} \end{array}$$

You try:

1. Determine the point and slope: $y + 3 = -2(x - 4)$
2. Write the equation of the line with a slope of $\frac{1}{3}$ that passes through the point (2, 4). Write your final answer in slope intercept form.

Answers: 1) point (-3, 4) slope: -2 2) $y = \frac{2}{3}x + \frac{10}{3}$

III. Standard Form

Linear equations can be written in several forms. One form is $Ax + By = C$. This is called standard form. You can find the slope and intercepts using the values of A, B, and C.

$Ax + By = C$
Slope is $-A/B$
x-intercept is C/A
y-intercept is C/B

Examples:

- a. Find the x and y intercepts of the following equation:

$$2x + 6y = 12$$

To find the x-intercept, set y equal to zero OR use C/A $C = 12$ $A = 2$
and solve for x.

$$2x + 6(0) = 12$$

$$12/2 = 6$$

$$2x = 12$$

$$x = 6$$

(6, 0) is the x-intercept.

To find the y-intercept, set x equal to zero OR use C/B $C=12$ $B=6$
and solve for y.

$$2(0) + 6y = 12$$

$$12/6=2$$

$$6y=12$$

$$y=2$$

(0,2) is the y-intercept.

- b. Write the equation in standard form using integers (no fractions or decimals).

$$y = \frac{-3}{4}x - 4$$

$$\frac{3}{4}x + y = -4 \quad \text{**in order to make the coefficient of x a whole number, multiply}$$

the entire equation by 4.

$$4\left(\frac{3}{4}x + y = -4\right) = 3x + 4y = -16$$

You try:

1. Find the x and y intercepts of the following equation: $14x - 7y = 7$
2. Write the equation in standard form using integers: $y = \frac{1}{2}x - 5$

Answers: 1) x: $(\frac{1}{2}, 0)$ y: $(0, -1)$ 2) $x - 2y = 10$ or $-x + 2y = -10$

Combining linear equations to find equations of lines

Write the equation of the line passing through the given points in standard form if you are given the points $(2, 3)$, $(-1, 5)$

First, Find the slope: $m = \frac{5-3}{-1-2} \quad m = -\frac{2}{3}$

Then, use the point-slope formula. It doesn't matter which point you pick, both points are on the line so either one will work.

Using the point $(2, 3)$: $y - 3 = -\frac{2}{3}(x - 2)$

Distribute and combine:

$$y - 3 = \frac{-2}{3}x + \frac{4}{3} \qquad y = \frac{-2}{3}x + \frac{13}{3}$$

Write in standard form (remember no fractions)

$$\frac{2}{3}x + y = \frac{13}{3} \quad \text{Multiply by 3}$$

Final answer: $2x + 3y = 13$

You try:

Write the equation of the line passing through the given points in standard form:

1. $(0, 3)$ $(-1, 4)$
2. $(-7, 6)$ $(-10, 4)$

Answers: 1) $x + y = 3$ 2) $-2x + 3y = 32$ or $2x - 3y = -32$