

### Objective 1.03 Model and Solve problems using direct variation

Direct Variation is represented by the equation  $y = kx$  where  $k$  is not zero. We say this as "y varies directly with x". This means that as y increases or decreases, the value of x increases or decreases.

The following example will explain direct variation in more detail:

Sam delivers newspapers each morning. He is paid \$0.25 for each paper he delivers. The following table relates the number of newspapers ( $x$ ) and his income ( $y$ ).

$x$	$y$
20	\$5.00
50	\$12.50
100	\$25.00
120	\$30.00
150	\$37.50

Sam's income depends directly on the number of newspapers he delivers.

- How much does he make for each newspaper he delivers?
- \$0.25 is called the constant of variation,  $k$ .
- The equation for the relationship between the number of newspapers and income is  $y = 0.25x$ .
- You can find the income,  $y$ , Sam will make for any number of newspapers he delivers.
  - How much would Sam make for delivering 250?
  - $y = 0.25(250)$  gives him \$62.50.

Let's say that we were not given how much Sam made for each newspaper, only the table.

How could you find how much he made for each newspaper?

You would divide the income by the number of newspapers he sold.

In other words to find the constant of variation, you would say  $k = \frac{y}{x}$ .

\*\*Since  $y = kx$  then  $k = \frac{y}{x}$  \*\*

Example 2:

The following table represents the number of video games you can buy with  $x$  amount of dollars. The number of video games you can buy varies directly with the number of dollars that you have.

Number of Dollars	30	60	90	120
Number of video games that can be purchased.	1	2	3	4

What is the constant of variation?

$$k = \frac{y}{x}$$

$$k = \frac{1}{30}$$

$$k = \frac{2}{60} = \frac{1}{30}$$

$$k = \frac{3}{90} = \frac{1}{30}$$

$$k = \frac{4}{120} = \frac{1}{30}$$

\*\* $k$  is the same throughout the table--another way of showing direct variation.\*

The constant of variation is  $\frac{1}{30}$ . From the table you may know that each game costs \$30. Do not be tempted to make that your  $k$ . For every one dollar you have, you can buy  $\frac{1}{30}$  of a game.

Write an equation for direct variation.

$$y = kx$$

$$y = \frac{1}{30}x$$

How many games can you buy with \$210?

$$y = \frac{1}{30}x$$

$$y = \left(\frac{1}{30}\right)(210)$$

$$y = 7$$

You can buy seven games with \$210

Example 3:

Write an equation of the direct variation that includes the given point.

1. (4, 16)

$$k = \frac{y}{x}$$

$$k = \frac{16}{4}$$

$$k = 4$$

$$y = 4x$$

2. (12, 2)

$$k = \frac{y}{x}$$

$$k = \frac{2}{12}$$

$$k = \frac{1}{6}$$

$$y = \frac{1}{6}x$$

3. (-5, 1)

$$k = \frac{y}{x}$$

$$k = \frac{1}{-5}$$

$$k = -\frac{1}{5}$$

$$y = -\frac{1}{5}x$$

Example 4:

The weight of an object on the moon varies directly with its weight on the earth. Susie weighs 168 pounds on earth and 28 pounds on the moon. Joe weighs 258 pounds on the earth. Find the constant of direct variation, write an equation, and find out how much Joe would weigh on the moon?

In the word applications for direct variation, find where it says

\_\_\_\_\_ varies directly with \_\_\_\_\_, and fill in the blanks. In this problem it says the weight of an object on the moon varies directly with its weight on earth.

Use the phrase  $y$  varies directly with  $x$ , to help you determine your  $y$  and  $x$ .  $Y$  is the weight on the moon, and  $x$  is the weight on earth.

Find  $k$ :

Weight on the moon/weight on the earth

28/168

$$k = \frac{1}{6} \quad \text{Equation: } y = \frac{1}{6}x$$

Joe weighs 258 on the earth. Substitute 258 in for  $x$  to give you his weight on the moon.

$$y = \frac{1}{6} (258) \quad y = 43$$

Joe would weigh 43 pounds on the moon.

Here are some for you to try:

1. Find the constant of variation and write the equation for the direct variation.

a.  $(21, 7)$

b.  $(\frac{1}{2}, 6)$

c.  $(5, \frac{2}{3})$

2. Suppose  $y$  varies directly as  $x$ , and  $y = 20$  when  $x = 4$ . Find  $y$  when  $x = 9$

3. For data in each table, tell whether  $y$  varies directly as  $x$ . If it does, write an equation for the direct variation.

a.

$x$	$y$
2	6
4	12
6	18
8	24

b.

$x$	$y$
1	2
2	5
3	6
4	10

4. Ken's wages vary directly as the time he works. If his wages for 4 days are \$110, how much will they be for 17 days?

Answers: 1) a.  $y = \frac{1}{3}x$  b.  $y = 12x$  c.  $y = \frac{2}{15}x$  2) 45 3) a. yes;  $y = 3x$  b. no

4) Ken would make \$467.50

