

## Objective 1.02 Solving Absolute Value Equations

**Absolute value** is the distance that a number is from zero on a number line.

For Example:  $|2| = 2$  and  $|-2| = 2$  Both 2 and -2 are two units away from zero.

To solve an absolute value equation:

- Isolate the absolute value.
- Set the expression inside the absolute value equal to the positive and negative values of the number on the right side of the equation.
- Solve the two equations.

**Example 1:** Solve:  $|x| = 3$       Solution:  $x = 3$  or  $x = -3$

The value of x can be 3 or -3 because  $|3| = 3$  and  $|-3| = 3$ .

**Example 2:** Solve:  $|x + 2| = 8$

Set up two equations and solve:  $x + 2 = 8$  or  $x + 2 = -8$   
$$\begin{array}{r} -2 \quad -2 \\ \hline x = 6 \end{array} \quad \text{or} \quad \begin{array}{r} -2 \quad -2 \\ \hline x = -10 \end{array}$$

$|6 + 2| = 8$  and  $|-10 + 2| = 8$

**Example 3:** Solve:  $\frac{1}{3}|2x - 3| + 5 = 10$

First isolate the absolute value:

$$\begin{array}{r} \frac{1}{3}|2x - 3| + 5 = 10 \quad \text{subtract 5} \\ \hline \frac{1}{3}|2x - 3| = 5 \\ \hline 3 * \frac{1}{3}|2x - 3| = 5 * 3 \quad \text{multiply by 3} \end{array}$$

$$|2x - 3| = 15$$

Set up two equations and solve:

$$\begin{array}{r} 2x - 3 = 15 \\ \hline +3 \quad +3 \\ \hline 2x = 18 \\ \hline \frac{2x}{2} = \frac{18}{2} \\ x = 9 \end{array} \quad \text{or} \quad \begin{array}{r} 2x - 3 = -15 \\ \hline +3 \quad +3 \\ \hline 2x = -12 \\ \hline \frac{2x}{2} = \frac{-12}{2} \\ x = -6 \end{array}$$

$$\frac{1}{3} |2(9) - 3| + 5 = 10 \quad \text{and} \quad \frac{1}{3} |2(-6) - 3| + 5 = 10$$

**Example 4:** Solve:  $\left| \frac{1}{2}x - 4 \right| + 10 = 3$

First isolate the absolute value:

$$\begin{array}{r} \left| \frac{1}{2}x - 4 \right| + 10 = 3 \\ \hline \phantom{\left| \frac{1}{2}x - 4 \right|} -10 \quad -10 \\ \left| \frac{1}{2}x - 4 \right| = -7 \end{array}$$

\*\*\*There is no solution. Absolute value can never be negative because it is the distance from zero and distance is not negative.\*\*\*

If you did not notice the answer is no solution, the answers you get will give you a false statement when you substitute them back in. This is shown below.

Set up your two equations and solve

$$\begin{array}{r} \frac{1}{2}x - 4 = -7 \quad \text{or} \quad \frac{1}{2}x - 4 = 7 \\ \hline \phantom{\frac{1}{2}x} +4 \quad +4 \qquad \phantom{\frac{1}{2}x} +4 \quad +4 \\ *2 \quad \frac{1}{2}x = -3 *2 \qquad \phantom{*2} 2 * \frac{1}{2}x = 11 *2 \\ \phantom{*2} x = -6 \qquad \phantom{*2} x = 22 \end{array}$$

$$\left| \frac{1}{2}(-6) - 4 \right| + 10 = 17$$

$$\left| \frac{1}{2}(22) - 4 \right| + 10 = 17$$

$$\left| \frac{1}{2}(-6) - 4 \right| + 10 \neq 3$$

$$\left| \frac{1}{2}(-6) - 4 \right| + 10 \neq 3$$

You Try:

1.  $|2k - 3| = 1$

2.  $|7 - 3p| = 5$

3.  $2|3x - 2| = 14$

4.  $7|5x| + 2 = 16$

5.  $|x + 1| + 5 = 3$

6.  $-|3x - 2| + 6 = 2$

Answers:

1.  $x = 2$  or  $x = 1$  2.  $p = \frac{2}{3}$  or  $p = 4$  3.  $x = 3$  or  $x = -\frac{5}{3}$  4.  $x = \frac{2}{5}$  or  $x = -\frac{2}{5}$

5. no solution 6.  $x = 2$  or  $x = -\frac{2}{3}$

